Controlling and Measuring Impedance in Variable Stiffness Robots

A.Bicchi[*+]

A. Ajoudani[*+], F. Belo[*], F. Bonomo[*], M. Catalano[*+], M. Gabiccini, M. Garabini[*], G. Grioli[*], M. Mancini[*], A. Passaglia[*], P. Salaris[*], P. Scilingo[*], G. Valenza[*], N. Tsagarakis[+]

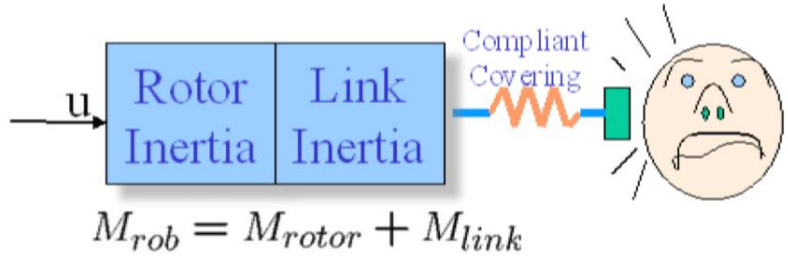
[*]Centro “E. Piaggio”, Università di Pisa

[+]IIT - Istituto Italiano di Tecnologia

Outline

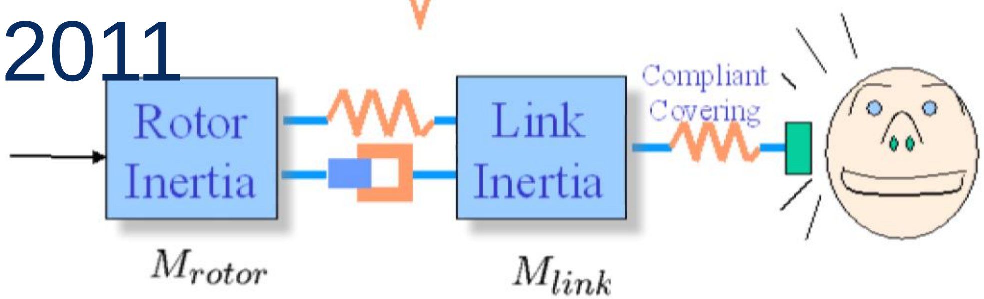
- A bit of a retrospective
- Using Variable Impedance
  - Optimal control
    - Safety oriented
    - Performance oriented
    - Energy oriented
  - Tele-Impedance
- Measuring Variable Impedance
- VSA and Hands
- Design

How to go beyond rigid robot limitations?

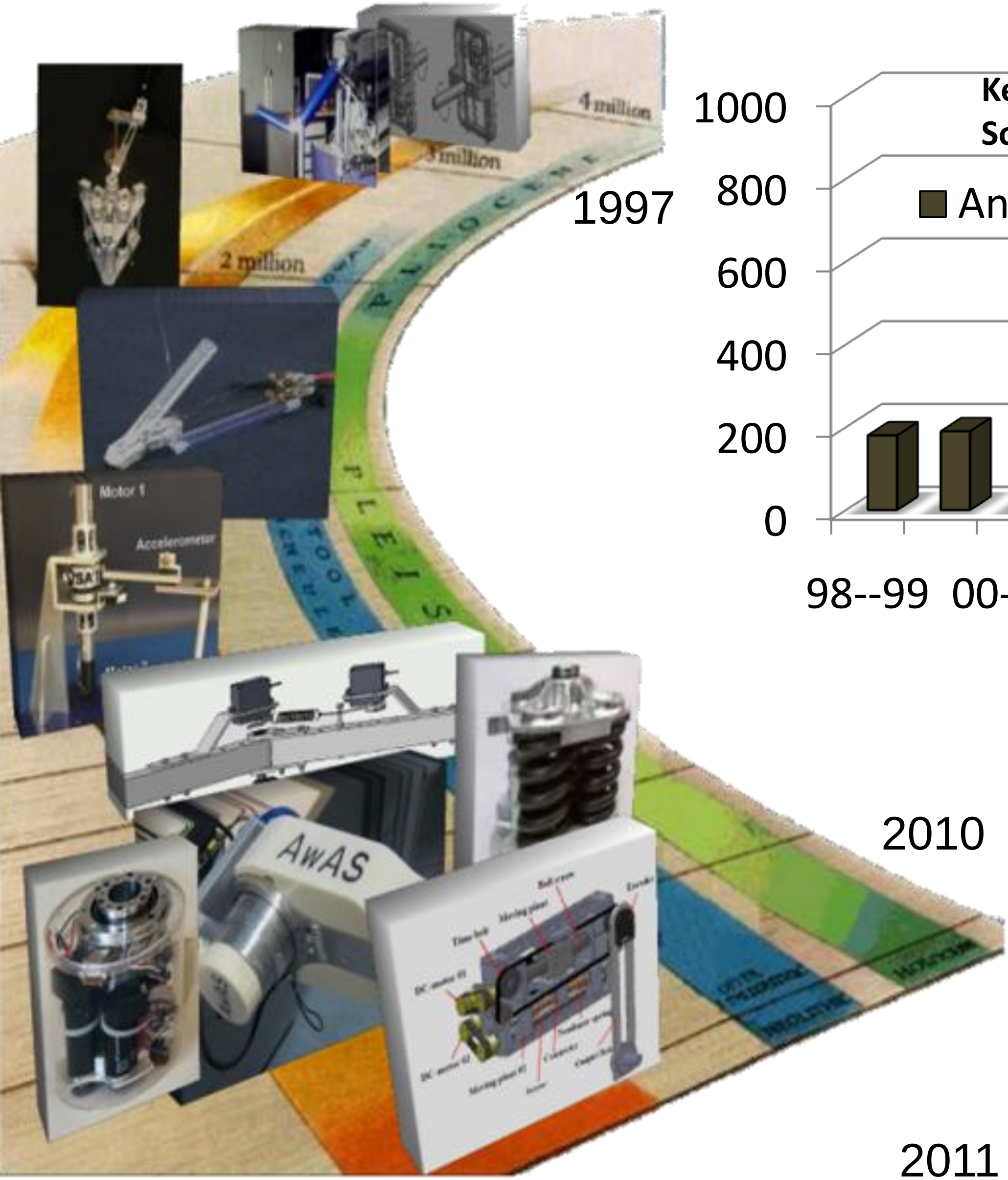


Basic idea:
decouple rotor inertia from link
via passive elasticity

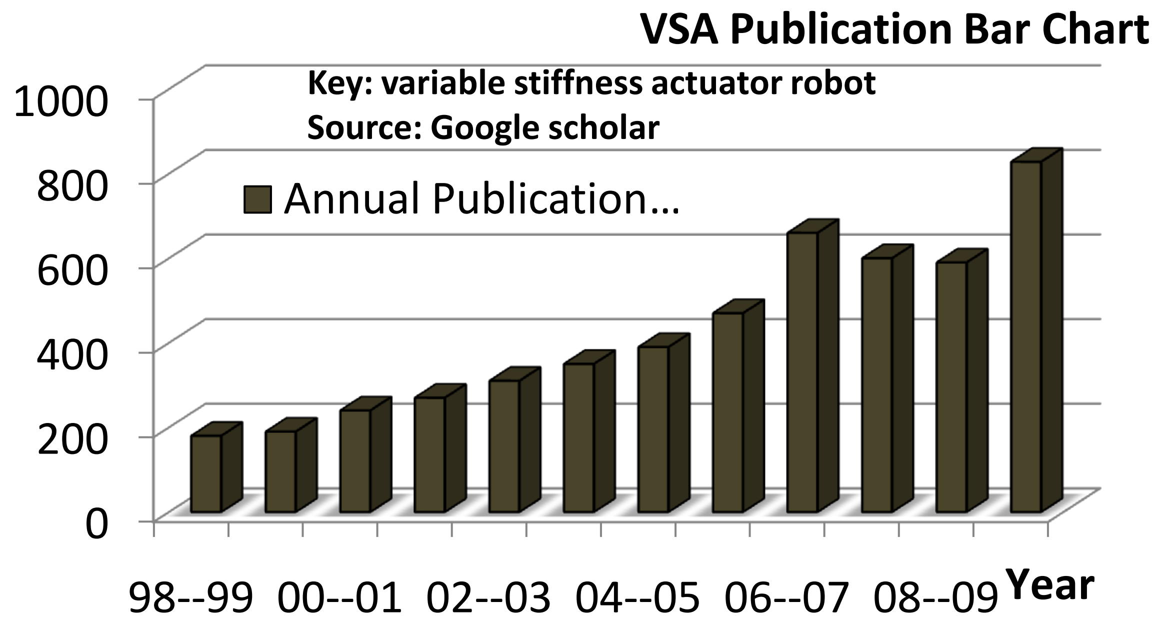
2011



Variable Stiffness Actuators



1997



2010 **European Projects:** PRHIENDS, VIACTORS, STIFF, THE, …

Newest **DARPA** call "M3" on variable compliance actuation US congress audition on "Safe, Soft Robots for Manufacturing"

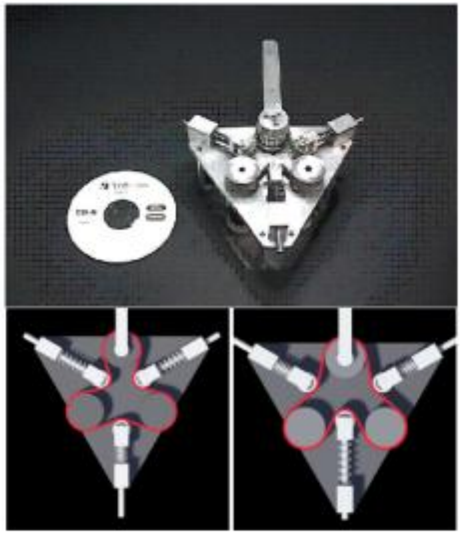
Heartland Robotics - "Robots to Empower American Workers"

2011 …

Variable Stiffness Actuators

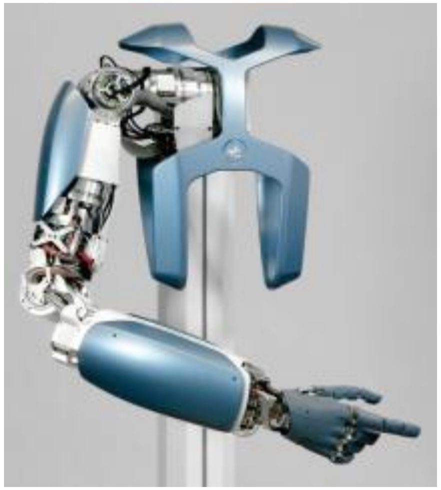
today

tomorrow



Pisa, 2003

Single DOF → multiDOF → applications



DLR, 2011

- locomotion
- physical H/R interaction
- manipulation
- haptics
- highly dynamic tasks
- unstructured environments
- ……….

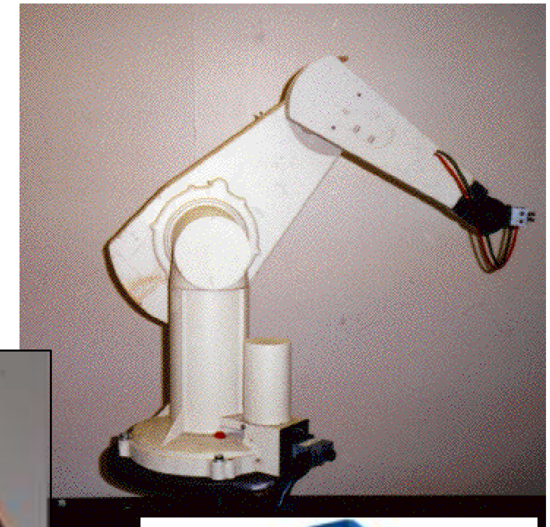
From *Motors* to *Muscles for Robots*

Robots vs. Humans
What a difference a body makes!

VSA as "Muscles for Robots"

What will future robots look like?

- From Position Control



- To Torque Control

- to Equilibrium Point
  AND
  Impedance Control

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  - Optimal control
    - Safety oriented
    - Performance oriented
    - Energy oriented
  - Tele-Impedance
- Measuring Variable Impedance
- VSA and Hands
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Optimal Control of Variable Impedance Actuators

M. Garabini[*], F. Belo[*], G. Grioli[*],
M. Mancini[*], A. Passaglia[*], P. Salaris[*]
A.Bicchi[*+]

[*]Centro “E. Piaggio”, Università di Pisa
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Safety-oriented Optimal Control

- Machines interacting with humans have different requirements than current in industry

  - ✓Accuracy less demanding
  - ✓Safety is a must

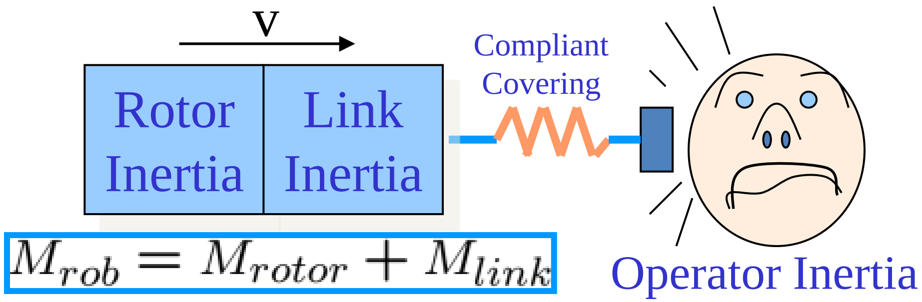
An approach: Design for Accuracy, Control for Safety

- ✓ Keep using rigid robots;
- ✓ Increase sensors drastically;
- ✓ Use active control;

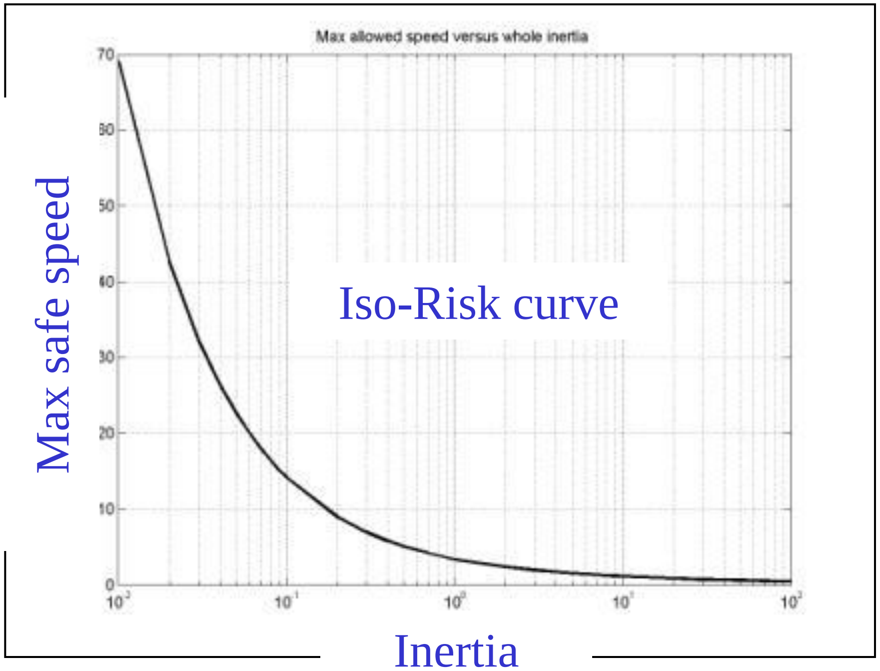
Modern approach: Design for Safety, Control for Accuracy

- ✓ Mechanical (passive) compliance;
- ✓ Compensation by control;

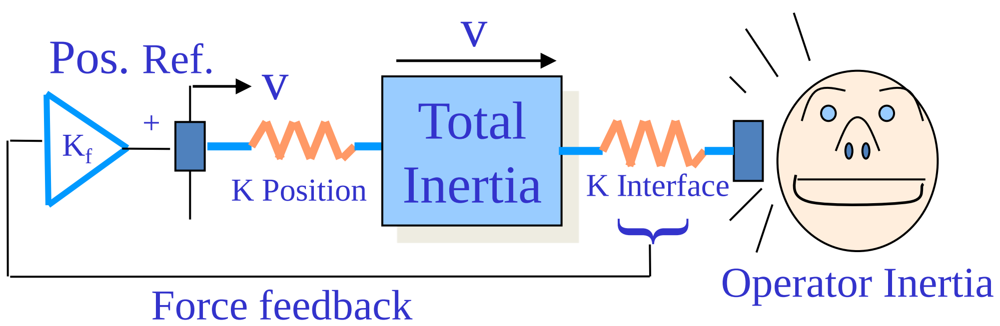
Conventional Design

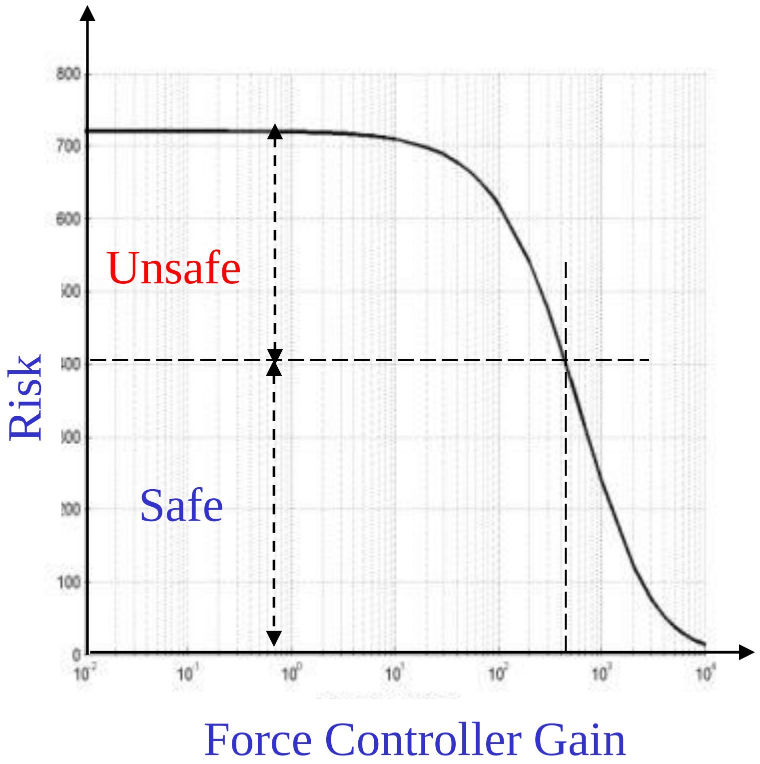


- First: minimize rotor + link inertia, use compliant covering if possible
- Given rotor + link inertia, covering, and acceptable risk → max. velocity
- Position control only makes this worse



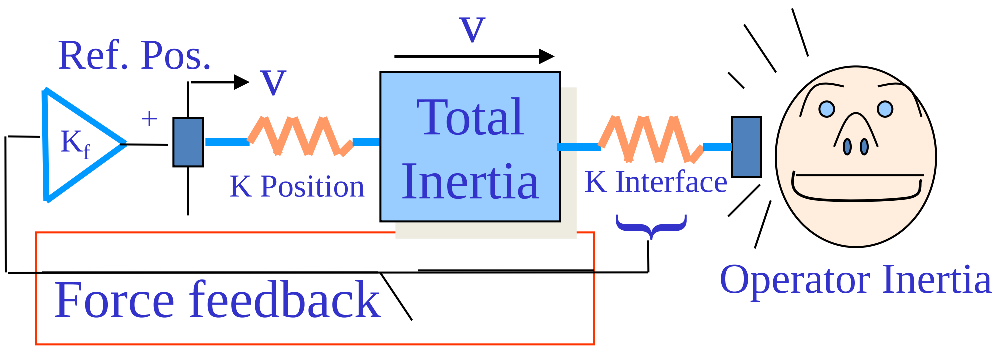
Conventional Design with Active Force Control - *Ideally*

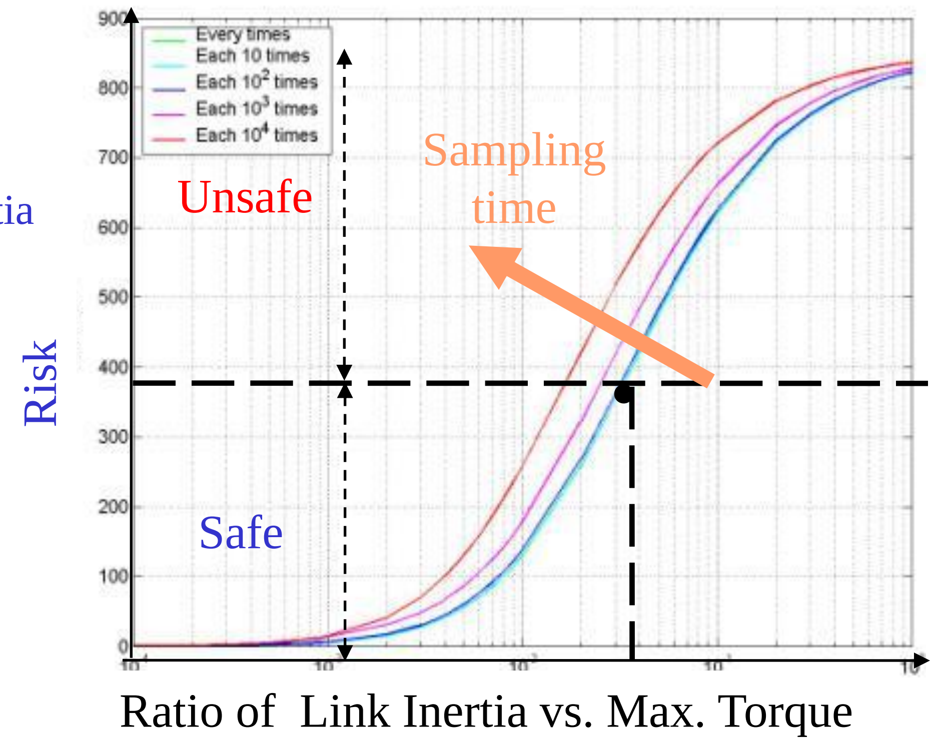




Theoretically, risk goes to zero as K_f increases.

Conventional Design with Active Force Control - *Really*

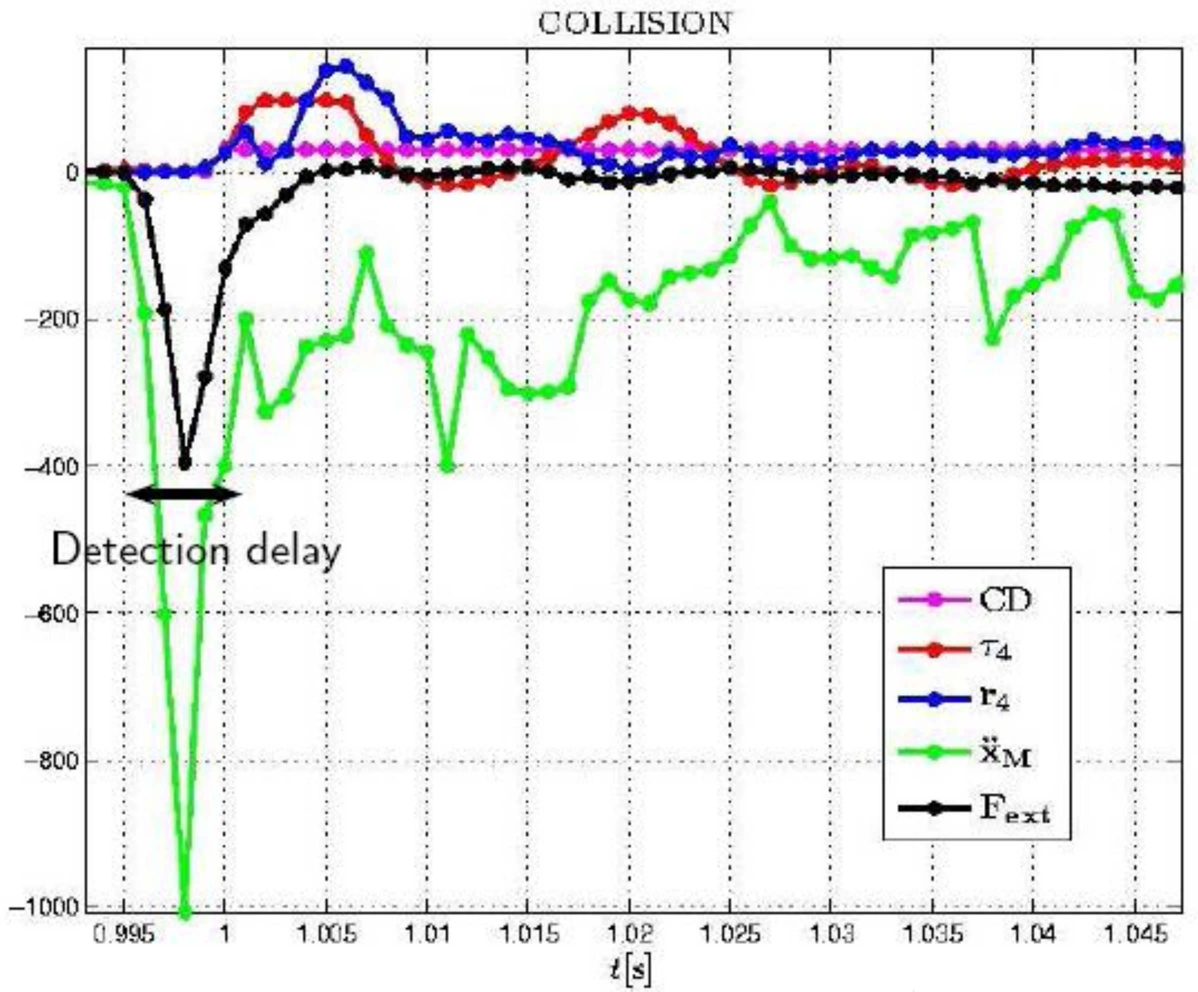




1) limited torque/link inertia ratio
2) limited mechanical bandwidth
3) limited sampling bandwidth

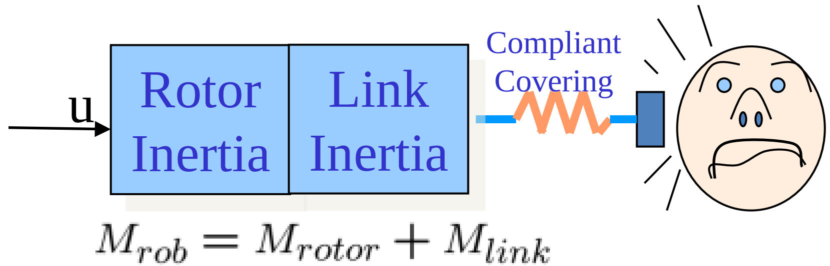
⟹ Active force control ineffective in real conditions

Conventional Design with Active Force Control - *Really*

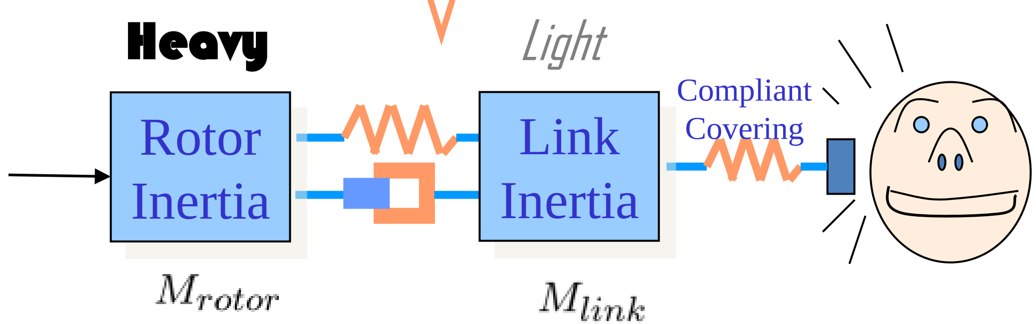


LWRII Data Courtesy DLR

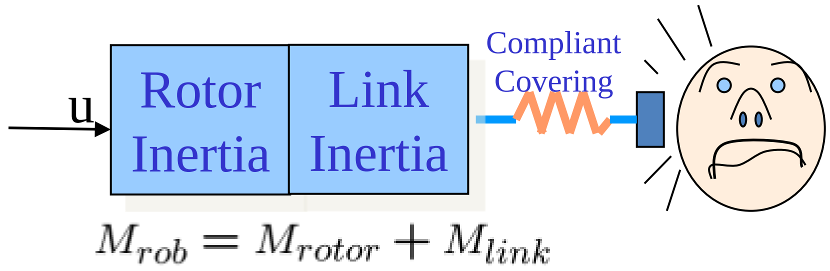
How to get beyond these limitations?



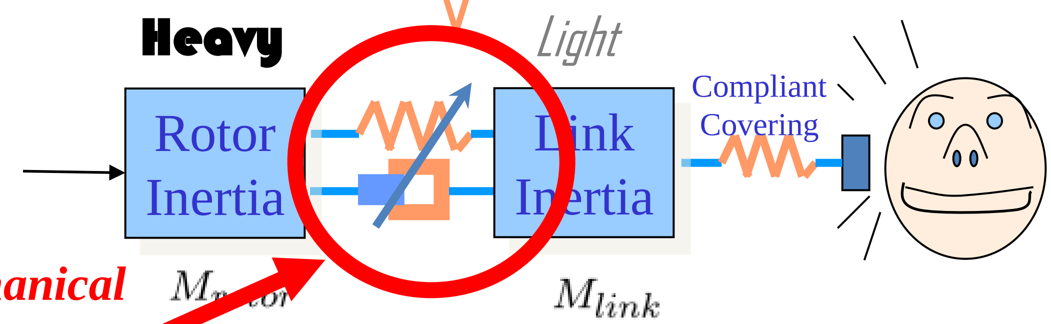
Basic idea:
decouple rotor inertia from link
via passive elasticity



How to get beyond these limitations? - 2



Basic idea:
decouple rotor inertia from link
via mechanical elasticity



Controllable Mechanical Compliance of the Joint

Soft Robotics

Constant Passive Compliance

- Compliance is fixed
- Absorb impact effects
- Adapt by changing the elastic element
- Only one motor
- Can be actively controlled to alter impedance

Series Elastic Actuators SEA

Variable Passive Compliance

- Compliance can be changed
- Adapt to situations, e.g.:
    - position a heavy load
    - move a glass of liquid
- Extra motor to alter impedance
- Increased complexity

Variable Stiffness Actuators VSA

Soft Robotics

SEA

VSA

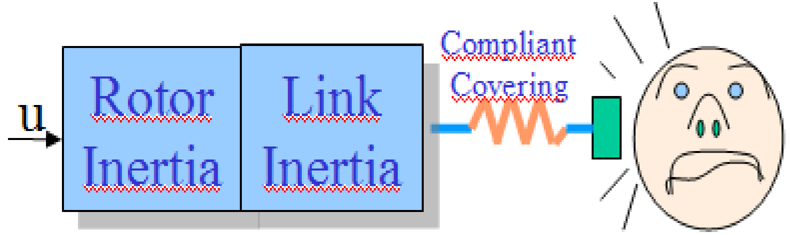
Slowly Variable Compliance

- Compliance can be changed once per task
- Adapt to nominally planned tasks
- Extra motor to alter compliance

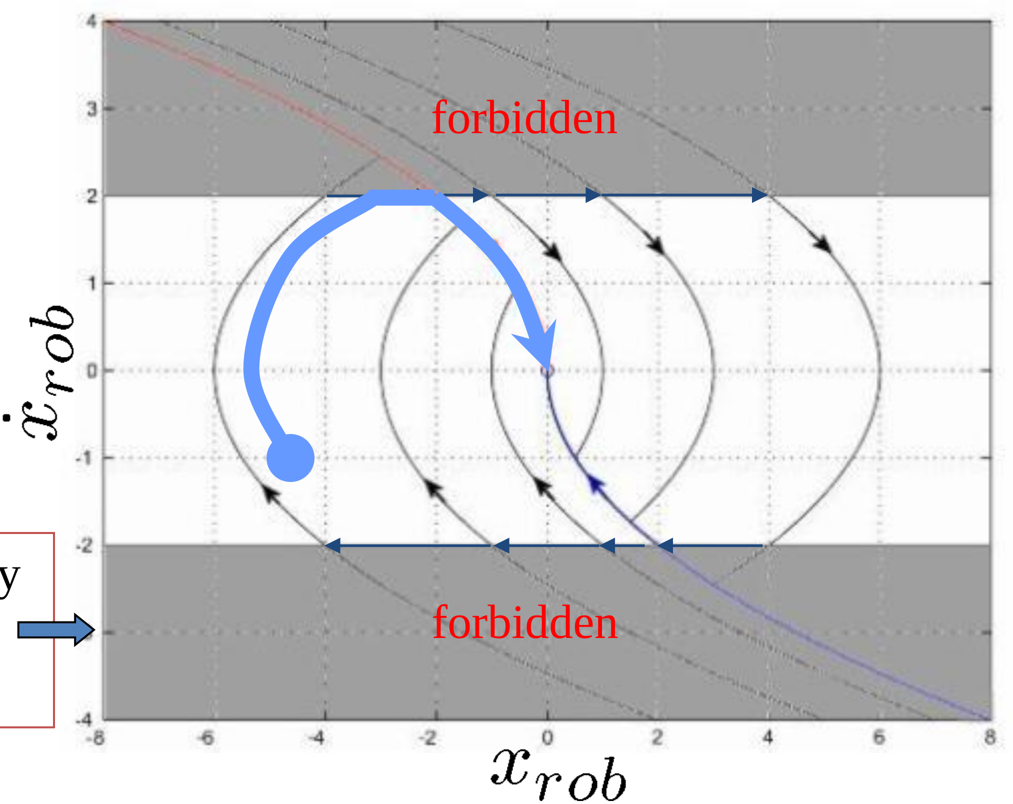
Quickly Variable Compliance

- Compliance can be changed with time constants comparable to motion
- Adapt stiffness *during* tasks, e.g.:
- (Larger) extra motor to alter compliance

The Safe Brachistocrone: Minimum Time Optimal Control with Safety

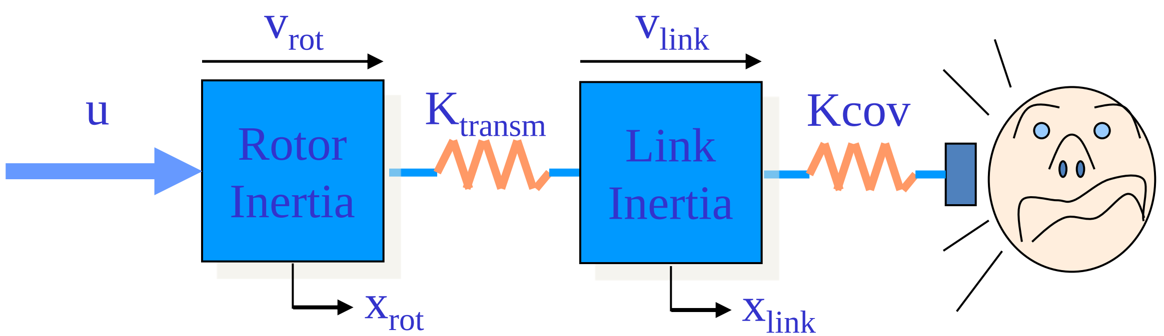


$$
\begin{cases}
\min_T \int_0^T 1 dt \\
M_{rob}\ddot{x}_{rob} = u \\
u \leq U_{max} \\
-u \leq U_{max} \\
v = \dot{x}_{rob} \leq \frac{HIC_{max}}{\beta} \\
-v = -\dot{x}_{rob} \leq \frac{HIC_{max}}{\beta}
\end{cases}
$$



The safe brachistochrone can be analytically solved in this case by applying Pontryagin's Maximum Principle

The Safe Brachistocrone for Series-Elastic Actuators



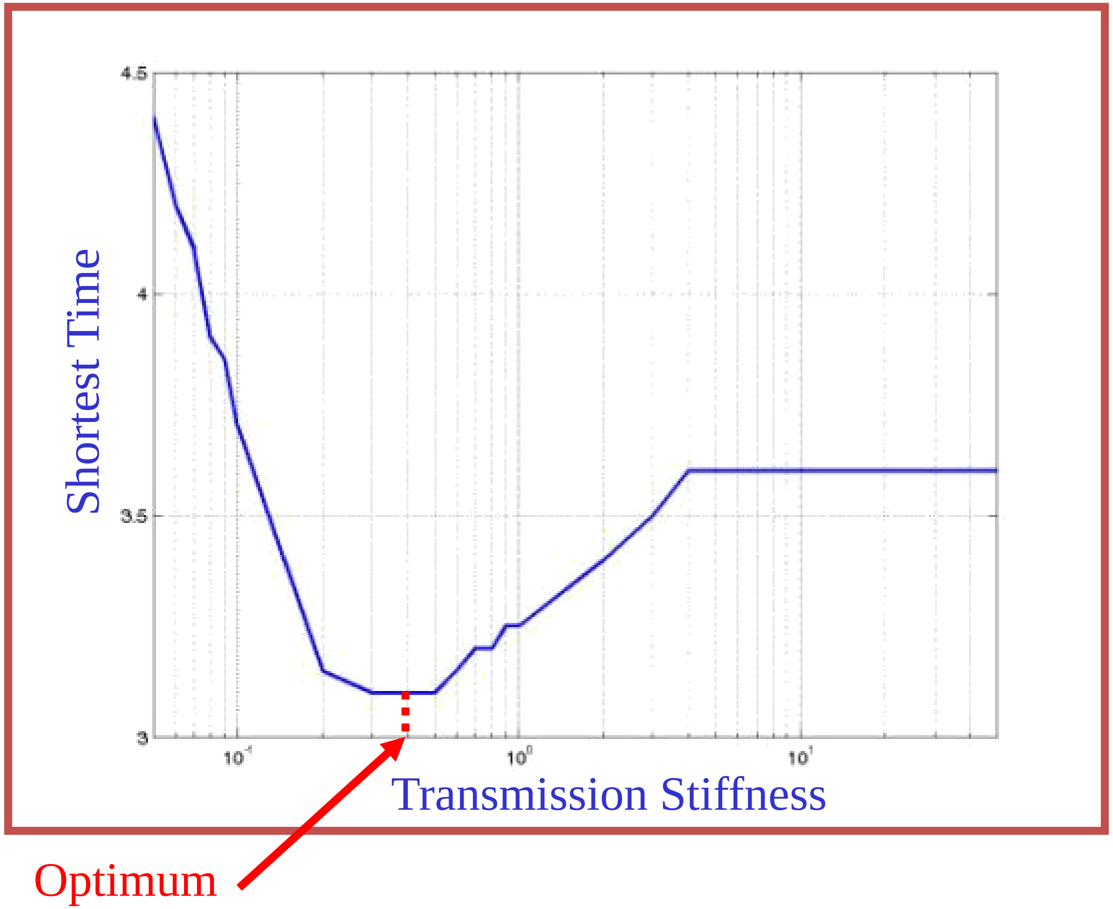
Linear system with linear inequality bounds

$$
\begin{cases}
\min_T \int_0^T 1 dt \\
M_{rot}\ddot{x}_{rot} + K_{transm}(x_{rot} - x_{link}) = u \\
M_{link}\ddot{x}_{link} + K_{transm}(x_{link} - x_{rot}) = 0 \\
|\dot{x}_{link}| \leq \beta HIC_{max}^{\frac{2}{5}} \\
|u| \leq U_{max} \\
(x_{link}, \dot{x}_{link})(0) = (X_{ini}, 0) \\
(x_{link}, \dot{x}_{link})(T) = (0, 0)
\end{cases}
$$

State & Control constraints (applies to $|\dot{x}_{link}| \leq \beta HIC_{max}^{\frac{2}{5}}$ and $|u| \leq U_{max}$)

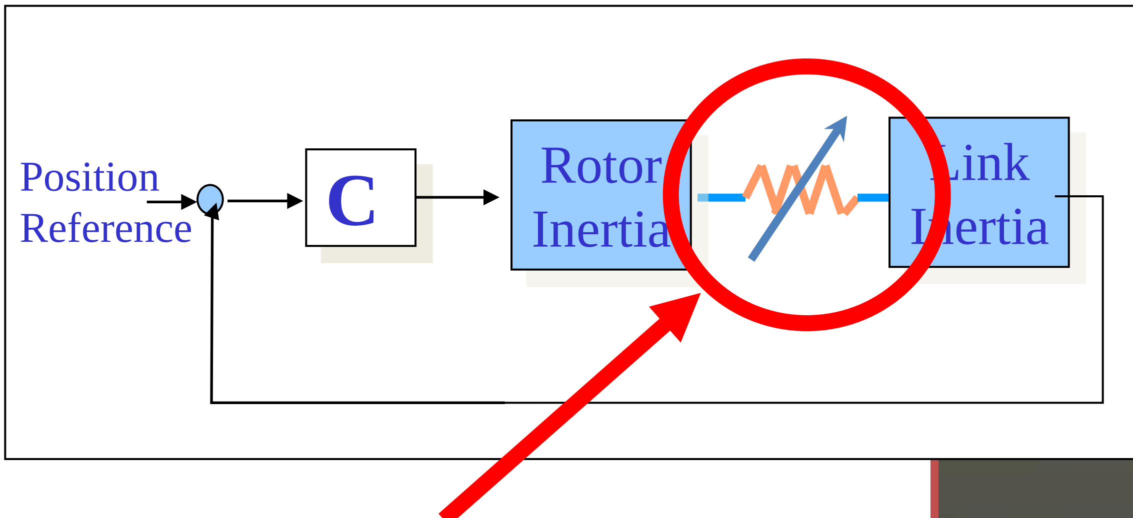
The Safe Brachistocrone for Series-Elastic Actuators

- For high transmission elasticity → slow response (oscillations, low accuracy,…);
- High transmission stiffness, high reflected inertia → low velocities for safety
- An optimum for transmission stiffness design exists
- Performance still limited

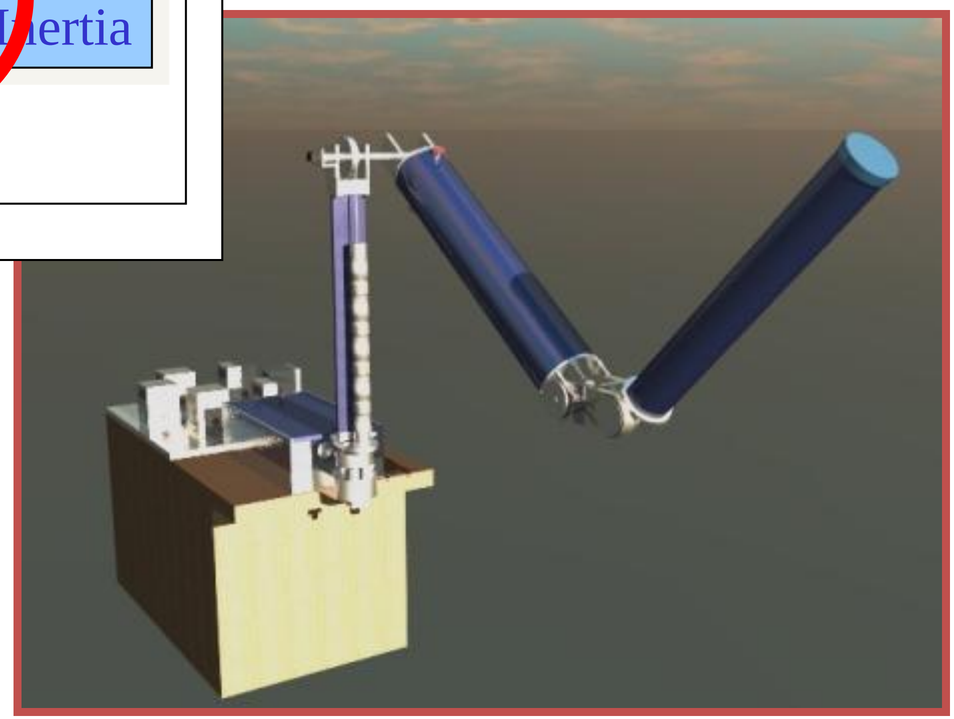


Recovering Performance by VSA

VST – Variable Stiffness Transmission
[Bicchi, Lodi Rizzini & Tonietti 2001]

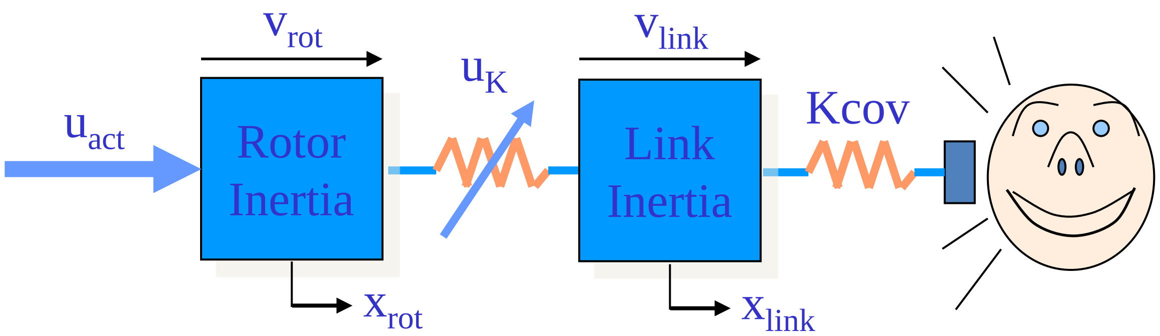


Controllable Mechanical Compliance of the Joint



Safe Brachistochrone for VSA

Can the control of transmission compliance recover performance?



The Safe Brachistocrone is now nonlinear

$$\begin{cases} \min_T \int_0^T 1 dt \\ M_{rot}\ddot{x}_{rot} + u_K(x_{rot} - x_{link}) = u_{act} \\ M_{link}\ddot{x}_{link} + u_K(x_{link} - x_{rot}) = 0 \\ |\dot{x}_{link}| \leq \beta(u_K) HIC_{max}^{\frac{2}{5}} \\ |u_{act}| \leq U_{max} \\ u_{K,min} \leq u_K \leq u_{K,max} \\ (x_{link}, \dot{x}_{link})(0) = (X_{ini}, 0) \\ (x_{link}, \dot{x}_{link})(T) = (0, 0) \end{cases}$$

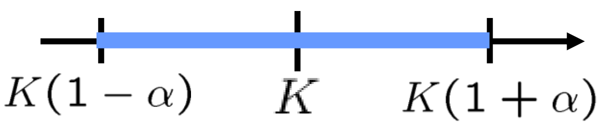
} Safety & Control bounds

VST Bounds

Comparison of VSA, SEA and DM²

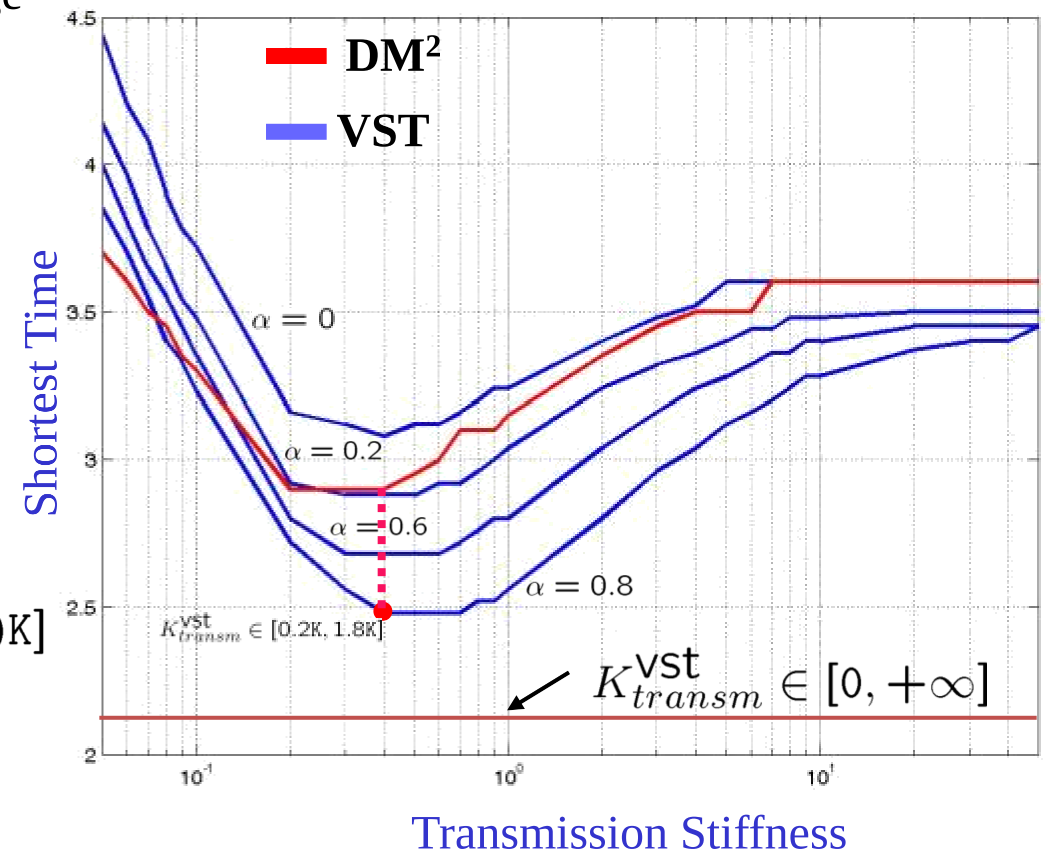
•VST performance recovery highest when transmission stiffness varies in broad range (ideally, 0 to ∞).

•Technology imposes bounds on range

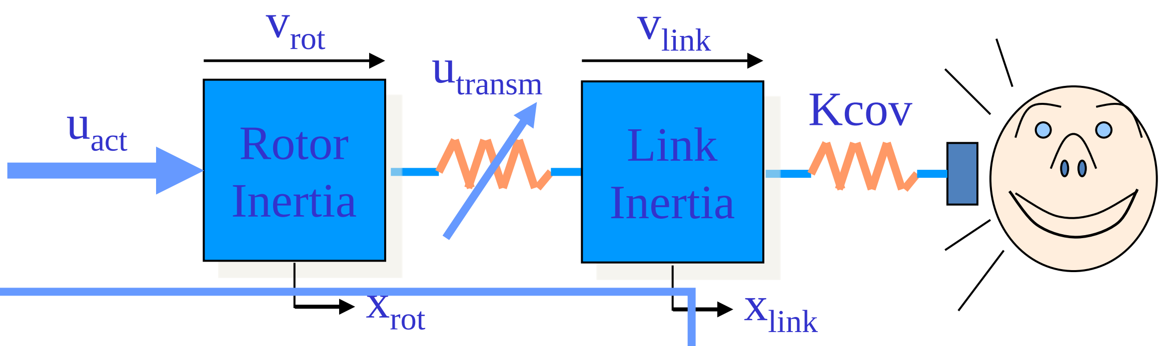


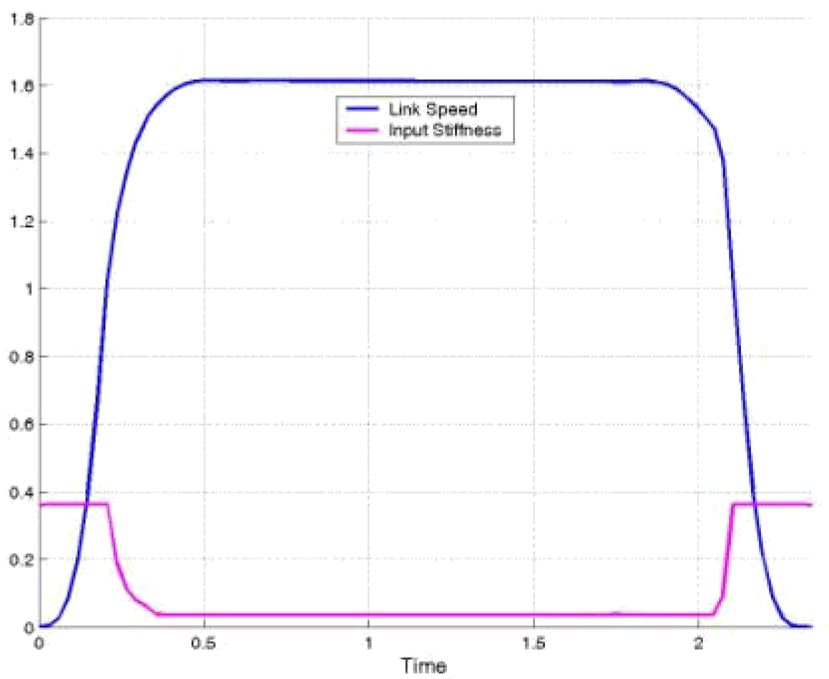
$$u_K = K_{transm}^{vst} \in [(1-\alpha)K, (1+\alpha)K]$$

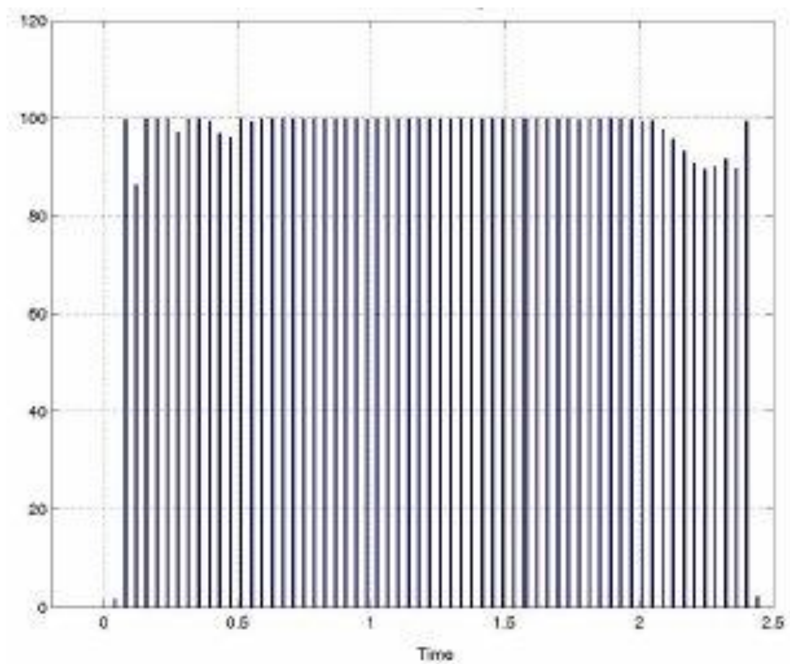
$$K = K_{transm}^{sea}$$



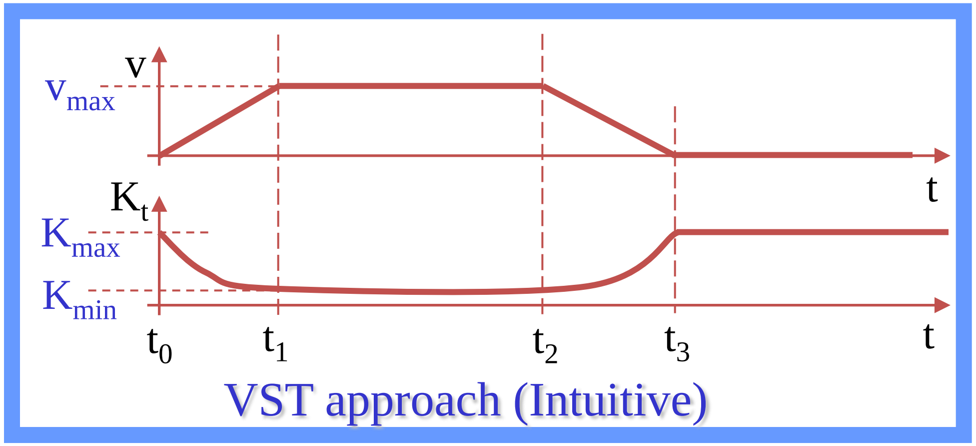
Simulation results for VSA (rotor & link)

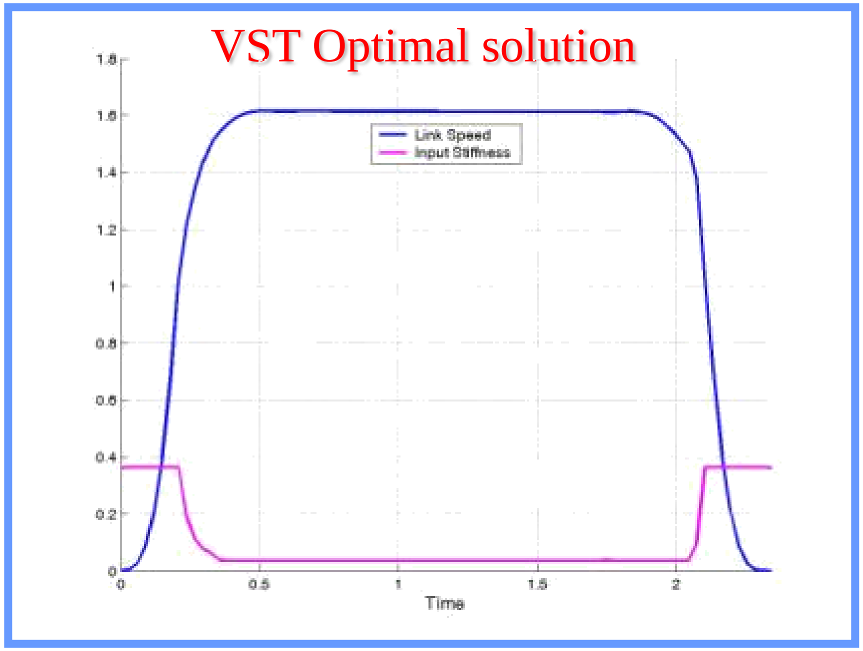






Control Policy for VSA





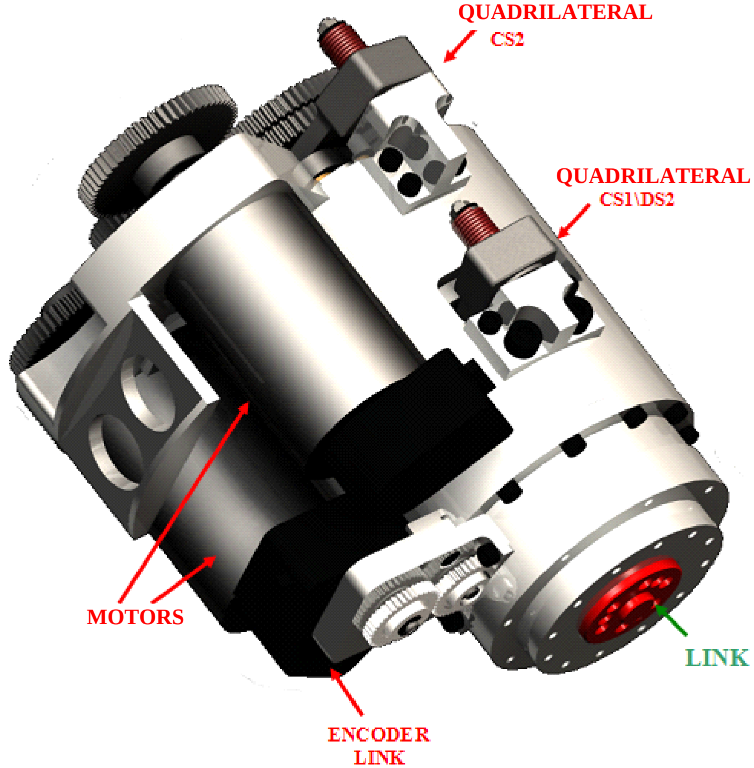
The intuitive policy of synchronizing joint stiffness and joint velocity is indeed consistent with the optimal solution for the safe brachistochrone!

→ Fast & Soft, Stiff & Slow

How to Plan Variable Impedance

- Optimal Control
  - Safe Brachistochrone
    ("soft and fast, stiff and slow")
  - Hit maximization (MaxSpeed problem)
  - Motion on a limit cycle (Energy Efficiency)
  - ...

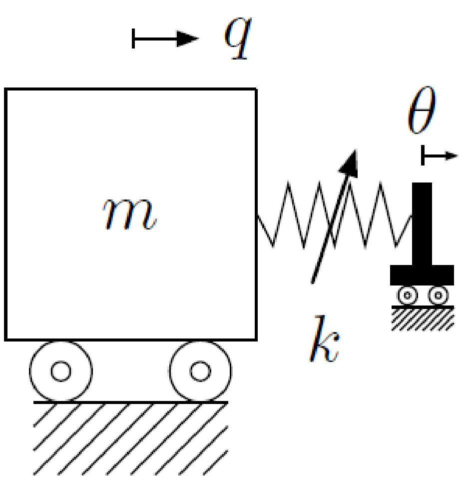
Exploiting the Dynamics of VSA



Drumming
Batting

Hammering

The MaxSpeed Problem
(The *VSA Hammer*)



$$|u| \leq u_{max}$$

Index $J = \phi(x(T)) = x_2(T) = \dot{q}(T)$.

Dynamics $\dot{x} = f(x,u)$,

Initial conditions $q(0) = 0$

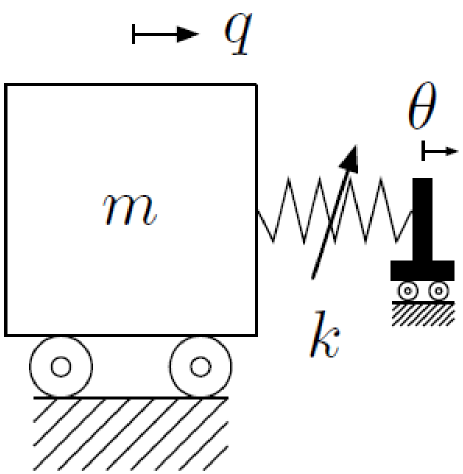
Terminal const. $\psi(x(T)) = x_1(T) = q(T) = 0$

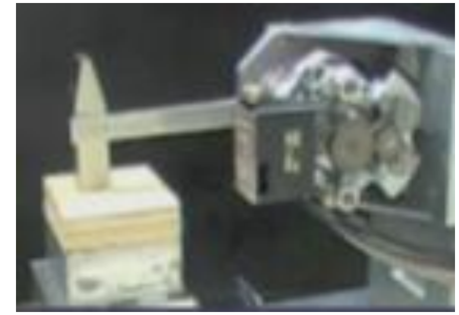
Hamiltonian $H(x(t),\lambda(t),u(t)) = \lambda^T(t) f(x(t),u(t))$

Optimality in Stiffness Control: Single Stroke MaxSpeed Problem

A. Passaglia, M.S. Thesis, 2010
M. Garabini, A. Passaglia, F. Belo, P. Salaris, A.B., IROS2011

The MaxSpeed Problem (The *VSA Hammer*)





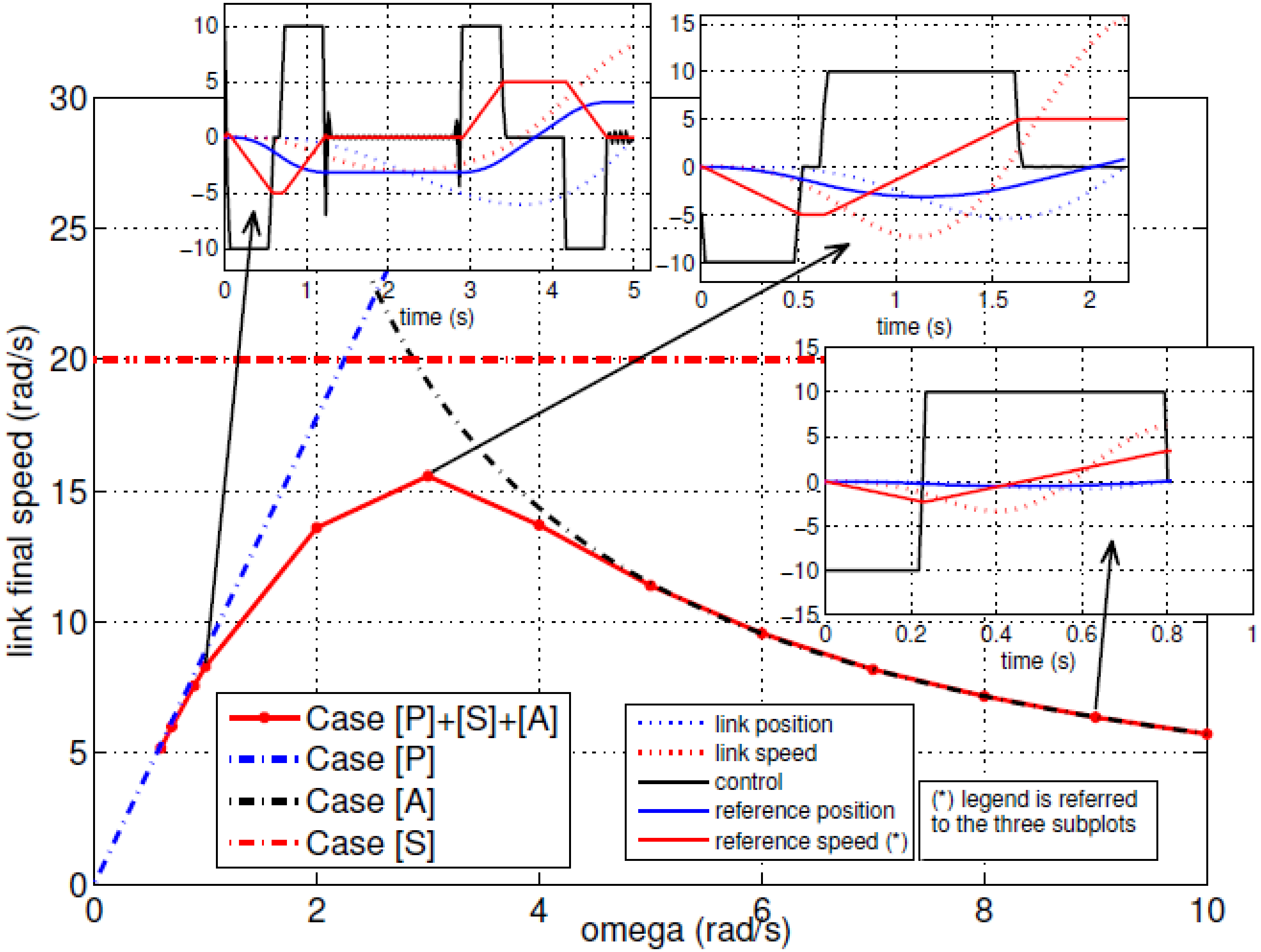
$$q(0) = 0$$
$$q(T) = 0$$

Constant K (SEA)

$$|u| \leq u_{max}$$
$$\omega = \sqrt{k/m}$$

Position control (P)	Speed control (S)	Acceleration control (A)
$\begin{cases} x^T = [q \quad \dot{q}] \\ u = \theta \\ \dot{x} = \begin{bmatrix} x_2 \\ \omega^2 (u - x_1) \end{bmatrix} \end{cases}$	$\begin{cases} x^T = [q \quad \dot{q} \quad \theta] \\ u = \dot{\theta} \\ \dot{x} = \begin{bmatrix} x_2 \\ \omega^2 (x_3 - x_1) \\ u \end{bmatrix} \end{cases}$	$\begin{cases} x^T = [q \quad \dot{q} \quad \theta \quad \dot{\theta}] \\ u = \ddot{\theta} \\ \dot{x} = \begin{bmatrix} x_2 \\ \omega^2 (x_3 - x_1) \\ x_4 \\ u \end{bmatrix} \end{cases}$
$v_{max} = 2\sqrt{2} u_{max} \omega$	$v_{max} = 4 u_{max}$	$v_{max} = 5.74 \frac{u_{max}}{\omega}$

Single Stroke MaxSpeed Problem:
Is There a Best Constant Stiffness?



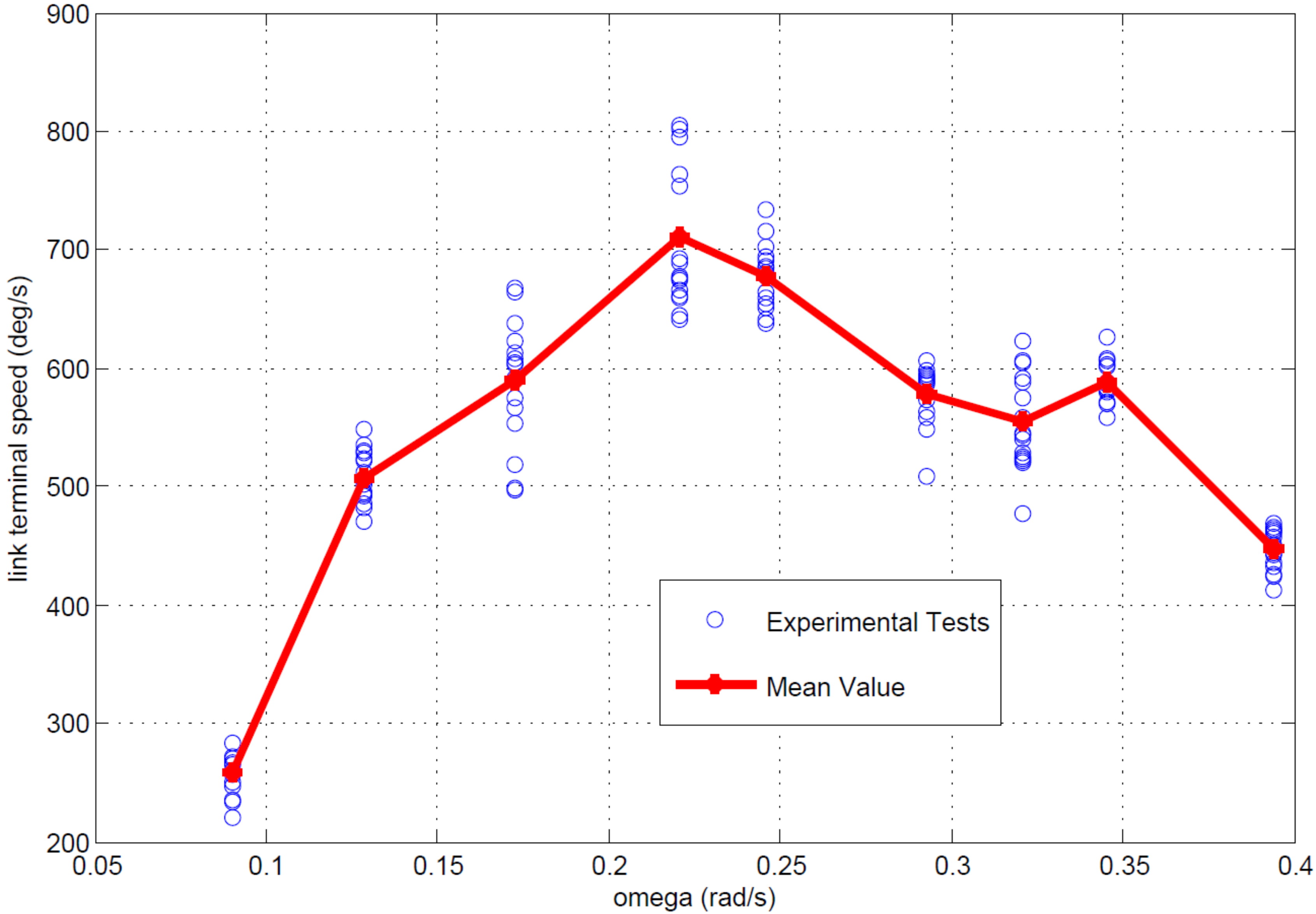
Position control: the stiffer, the better

Velocity control: *k*-invariant

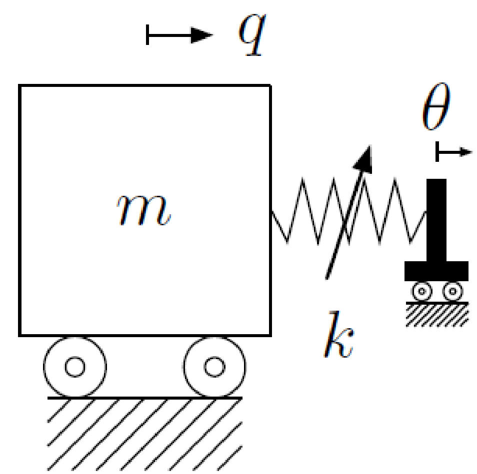
Acceleration control: the softer, the better

Realistic Conditions (Acc.Ctrl. s.t. Pos., Speed constraints) → *k^{opt} exists!*

Experimental Results



MaxSpeed Problem: Can VSA improve further?



$$
\begin{cases}
x^T = [q \quad \dot{q}] \\
u^T = [\theta \quad k] \\
\dot{x} = \begin{bmatrix} x_2 \\ \frac{u_2}{m}(u_1 - x_1) \end{bmatrix}
\end{cases}
$$

State space definition	$\begin{cases} x^T = [q \quad \dot{q}] \\ u^T = [\theta \quad k] \\ \dot{x} = \begin{bmatrix} x_2 \\ \frac{u_2}{m}(u_1 - x_1) \end{bmatrix} \end{cases}$
Hamiltonian	$H = \lambda_1 x_2 - \lambda_2 \frac{u_2}{m}(x_1 - u_1)$
Co-State dynamics	$\begin{cases} \dot{\lambda}^T = \begin{bmatrix} \frac{u_2}{m}\lambda_2 & -\lambda_1 \end{bmatrix} \\ \lambda(T)^T = [0 \quad 1] \end{cases}$
Optimal control law	$u_1^* = \begin{cases} u_{1,max} & \text{if } \lambda_2 > 0 \\ -u_{1,max} & \text{if } \lambda_2 < 0 \end{cases}$ $u_2^* = \begin{cases} u_{2,max} & \text{if } \lambda_2(u_1 - x_1) > 0 \\ u_{2,min} & \text{if } \lambda_2(u_1 - x_1) < 0 \end{cases}$

Analytical solution available! (IROS 2011)

Main Results (IROS 2011)

Theorem 1: The optimal control is characterized by the following properties:

1) the switching sequence is

$$\{S_2; S_{1,2}; S_2; \ldots; S_2; S_{1,2}\}\,, \tag{14}$$

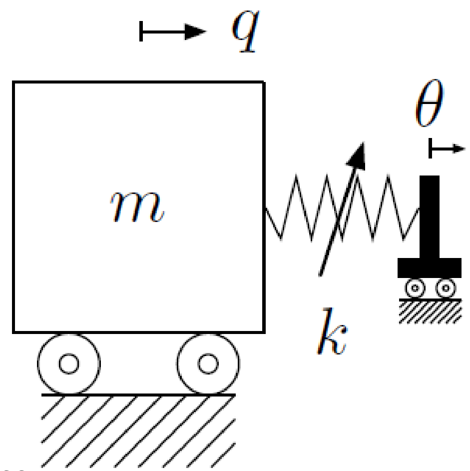
2) the time between S_2 and $S_{1,2}$ is

$$t_{S_{1,2}} = \sqrt{m/k_{min}}\pi/2\,, \tag{15}$$

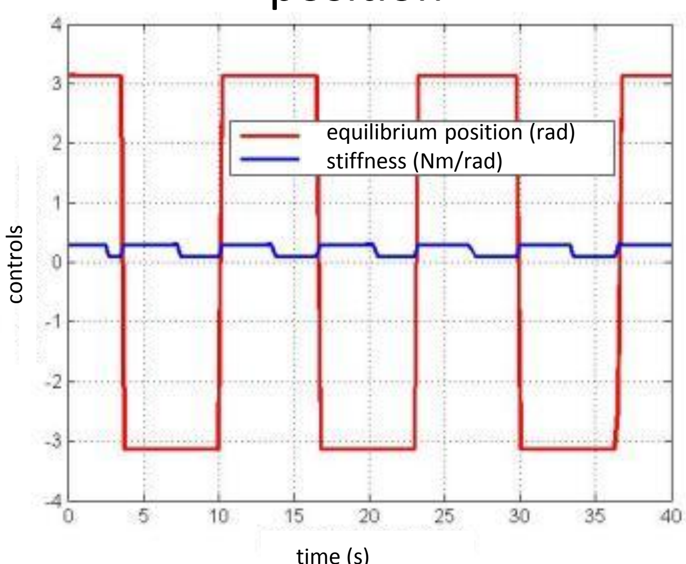
3) the time between $S_{1,2}$ and S_2, the time of the first period and the time of the last period are:

$$t_{S_2} = \sqrt{m/k_{max}}\pi/2\,, \tag{16}$$

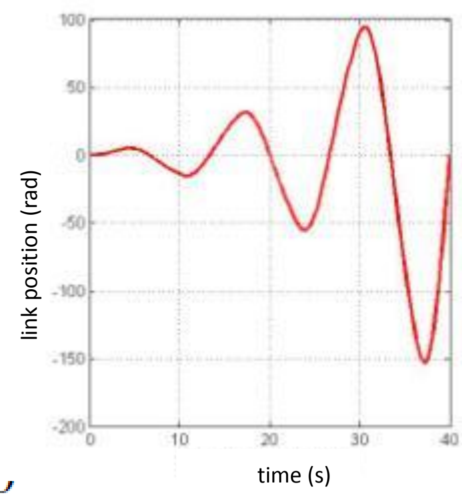
MultiStroke MaxSpeed Problem



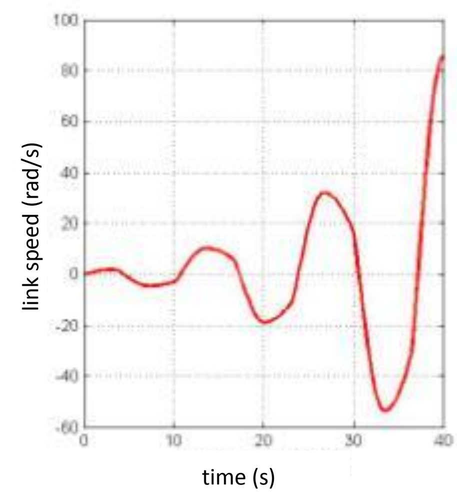
Optimal stiffness and equilibrium position



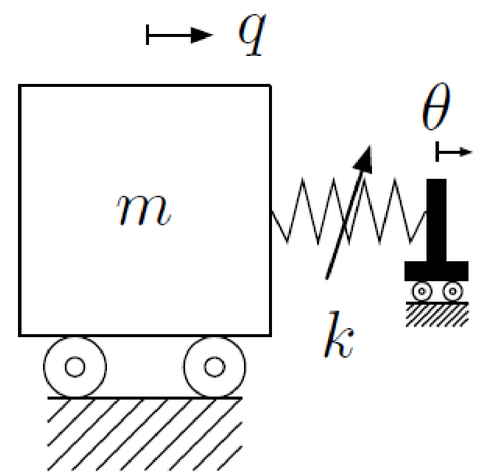
Joint position Vs time



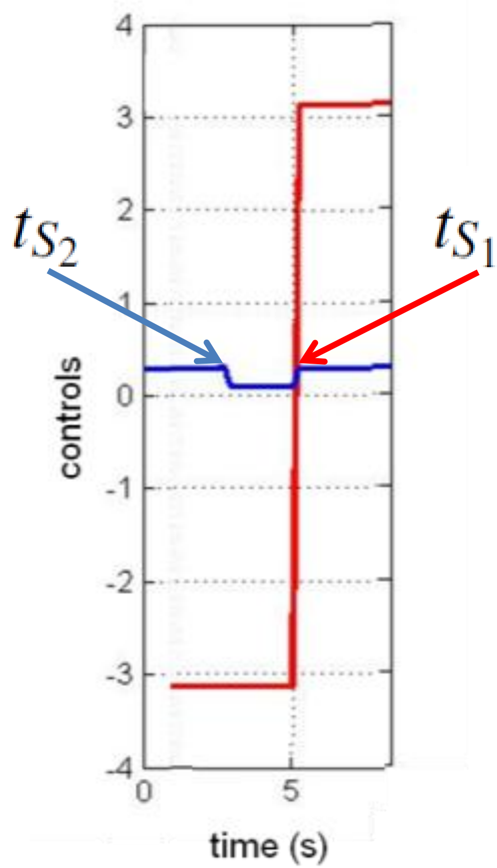
Joint speed Vs time

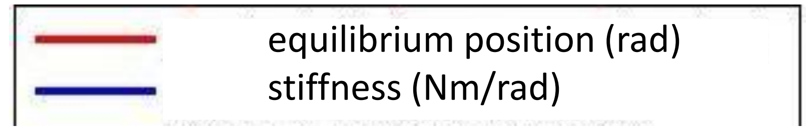


Single Stroke MaxSpeed Problem: Can VSA improve further?



$$\begin{cases} x^T = [q \quad \dot{q}] \\ u^T = [\theta \quad k] \\ \dot{x} = \begin{bmatrix} x_2 \\ \frac{u_2}{m}(u_1 - x_1) \end{bmatrix} \end{cases}$$





$$t_{S_1} = \sqrt{m/k_{min}}\pi/2$$

$$t_{S_2} = \sqrt{m/k_{max}}\pi/2$$

t_{S_2} link acceleration changes sign

t_{S_1}: link velocity changes sign

$$\frac{v_{VSA}}{v_{SEA}} = \frac{1 + (k_{max}/k_{min})^{1/2}}{2}$$

Yes, up to 30% in our setup !

Note: position controlled SEA with optimal (max) K

Main Results (IROS 2011)

Theorem 1: The optimal control is characterized by the following properties:

1) the switching sequence is

$$\{S_2; S_{1,2}; S_2; \ldots; S_2; S_{1,2}\}, \tag{14}$$

2) the time between S_2 and $S_{1,2}$ is

$$t_{S_{1,2}} = \sqrt{m/k_{min}}\pi/2, \tag{15}$$

3) the time between $S_{1,2}$ and S_2, the time of the first period and the time of the last period are:

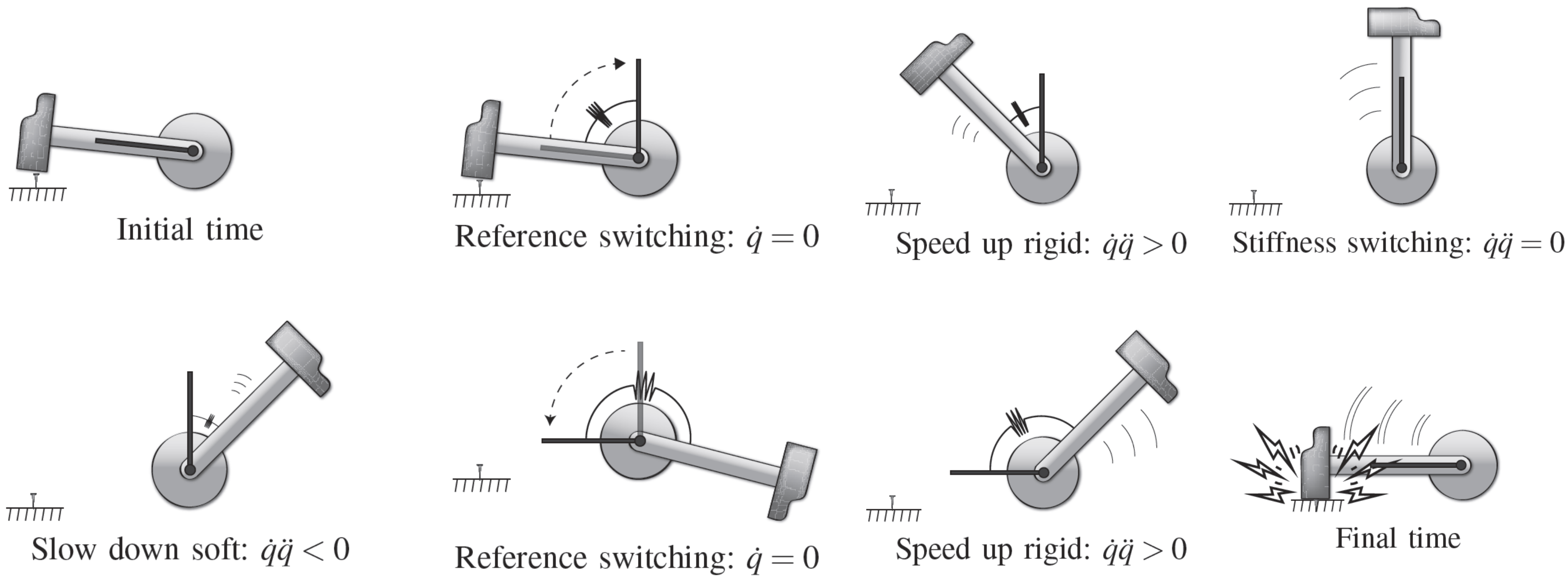
$$t_{S_2} = \sqrt{m/k_{max}}\pi/2, \tag{16}$$

Theorem 2: The stiffness optimal control is:

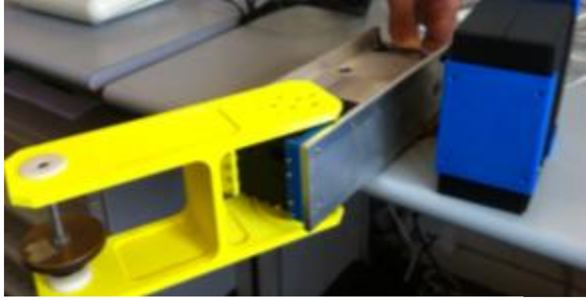
$$u_2 = \begin{cases} u_{2,\max} & \text{if } \dot{q}\ddot{q} > 0 \\ u_{2,\min} & \text{if } \dot{q}\ddot{q} < 0 \end{cases}$$

→ Stiff speed-up, soft slow-down

VSA Optimal control

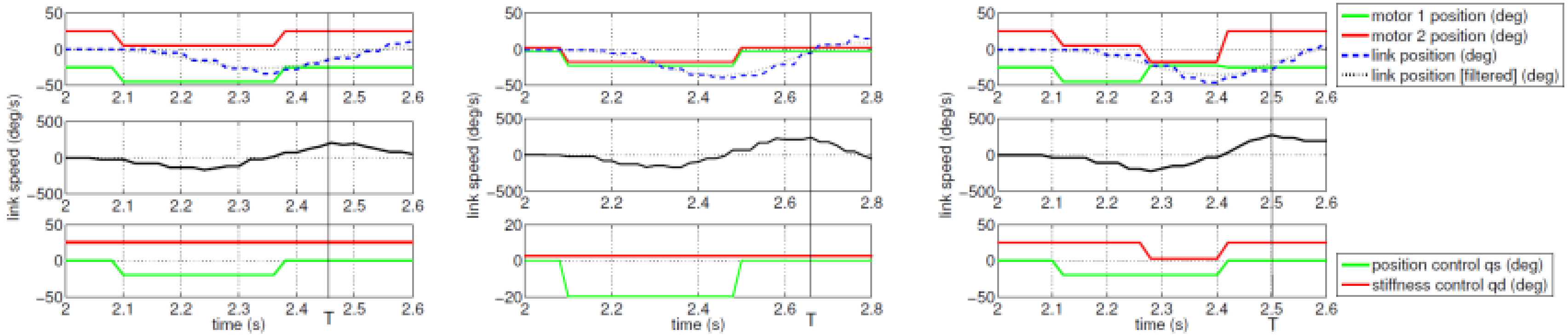


Single Stroke MaxSpeed Problem: Experimental



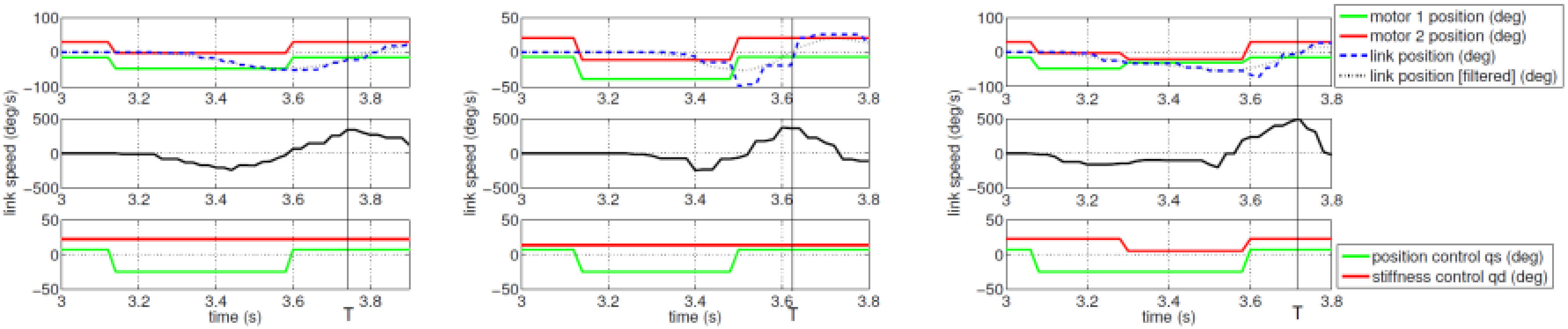
Max. stiffness **Min. stiffness** **Variable stiffness**

Light mass



(a) $m = 0.1\text{kg}$, $q_{D,max}$, $\dot{q}(T) \simeq 207\text{deg/s}$ (b) $m = 0.1\text{kg}$, $q_{D,min}$, $\dot{q}(T) \simeq 242\text{deg/s}$ (c) $m = 0.1\text{kg}$, $\dot{q}(T) \simeq 276\text{deg/s}$

Heavy mass



(d) $m = 1\text{kg}$, $q_{D,max}$, $\dot{q}(T) \simeq 341\text{deg/s}$ (e) $m = 1\text{kg}$, $q_{D,mid}$, $\dot{q}(T) \simeq 374\text{deg/s}$ (f) $m = 1\text{kg}$, $\dot{q}(T) \simeq 499\text{deg/s}$

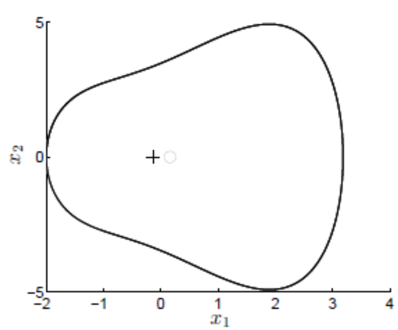
Optimization of Cyclic Movements

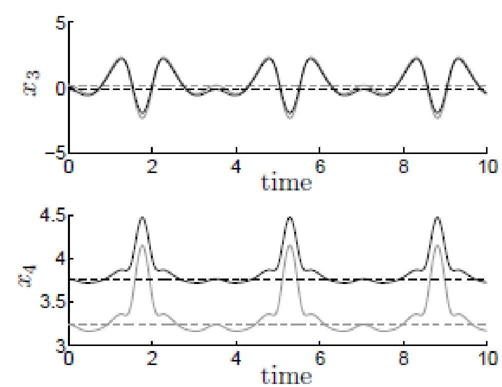
Embodying Desired Behavior in Variable Stiffness Actuators ⋆

IFAC WC 2011

Ludo C. Visser * Stefano Stramigioli * Antonio Bicchi **

- Mechanical compliance introduces intrinsic, passive oscillatory behavior: you can fight this, or exploit it
- Concept of nonlinear resonant modes
- VSA can be controlled such that its passive behavior is as close as possible to the desired behavior and the control effort is minimized.
- The cost criterion provides a **measure of embodiment** of the desired behavior in the passive behavior of the variable stiffness actuator



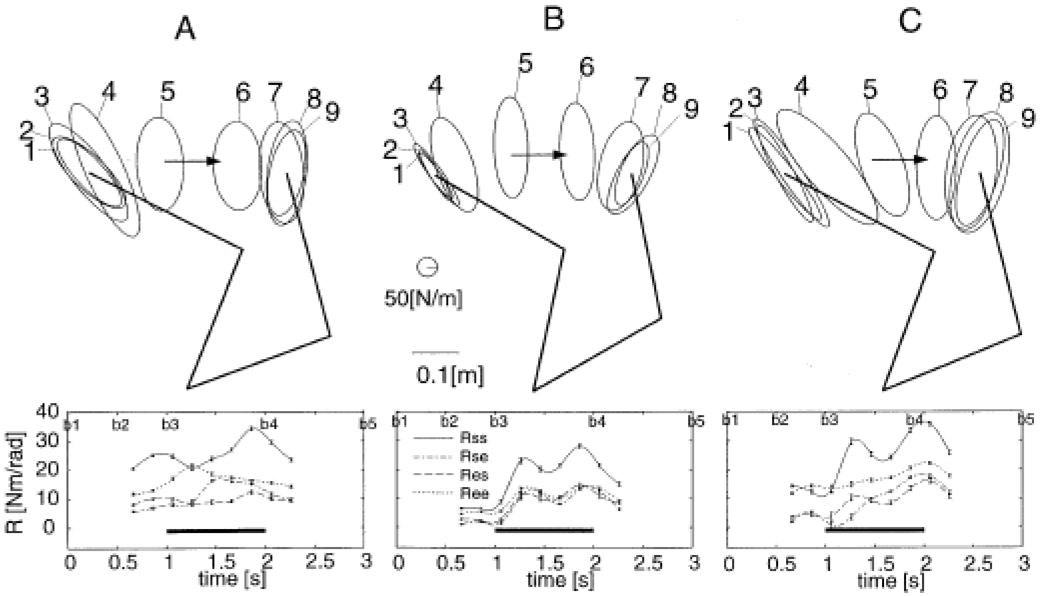


Outline

- A bit of a retrospective
- Using Variable Impedance
  - Optimal control
    - Safety oriented
    - Performance oriented
    - Energy oriented
  - Tele-Impedance
- Measuring Variable Impedance
- VSA and Hands
- Design

How to Plan Variable Impedance

- How do humans use variable impedance?
  - study human subjects
    - Arm stiffness is adapted to tasks and force fields [Kawato, Hogan, Burdet, Gomi, Franklin, Shadmer, …]
    - Results look at impedance in one configuration after several learning trials
    - How do humans vary their stiffness while throwing, drumming, walking ?



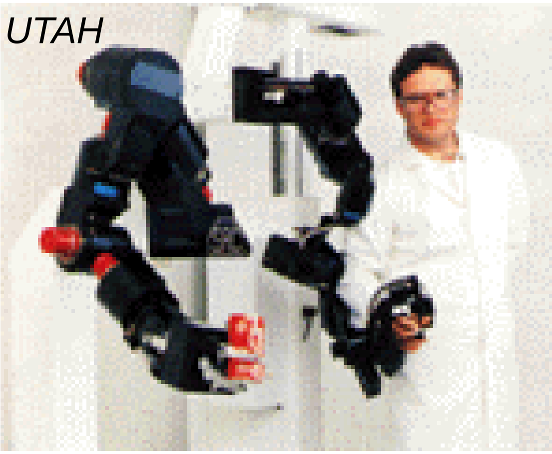
How to Plan Variable Impedance

- Let the human do it
  - Not general
  - Makes sense in applications where you need to replicate some skills at a distance

→ Teleoperation of Impedance

Teleoperation

- Teleoperation consists of measuring motor control parameters in human, and replicating in robots
- Remote position control is rather easy – used to be done with exoskeletons as “master” and robot arms as “slave”
- Position control is not enough because of interaction with unknown remote environments



Teleimpedance

- Remote control with force feedback
  - needs force sensors on the robot
  - may have stability problems because of transmission delays in the control loop
- Can we instead do remote impedance control?
  - use human arm position and impedance as references for robot
  - use local controllers (no delays, very stable) to make the robot track those plans

Teleimpedance

- The operator moves the hand and adapts his/her arm impedance to solve the task based on visual information from the scene

Three questions:

- How do we sense position and impedance on the master?
- How do we control them on the slave?
- Is this useful to solve the task?

EMG

- Electromyographic signals correspond to muscle activation, hence are strongly correlated with muscle force
- Very easy to implant, not invasive, relatively cheap (compared to force-reflecting exoskeletons)
- The command signal of choice for prostheses
- Signals are noisy and not too stable – good processing needed

EMG and Robotics

- Sophisticated signal processing techniques should be applied for precise continuous estimation of the movement of the human limbs, because of
  - cross-talk between channels of surface EMGs
  - correlation between EMGs and isometric muscle tension and impedance
  - Typical EMG processing is by segments, introducing latency which affects negatively real-time control of a robot

EMG and Robotics

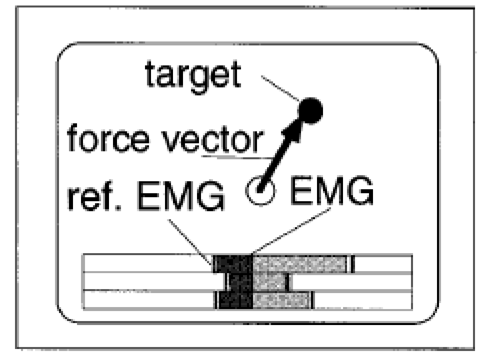
- A significant amount of research in applications of EMGs in Robotics, exoskeletons and prosthetic devices
- Few among many notable contributions:
  - B. E. Mustard and R. G. Lee (1987);
  - Castellini and van der Smagt (2008-2009);
  - Bu, Okamoto, and Tsuji (2009);
  - Artemiadis and Kyriakopoulos (2010),
  - ...

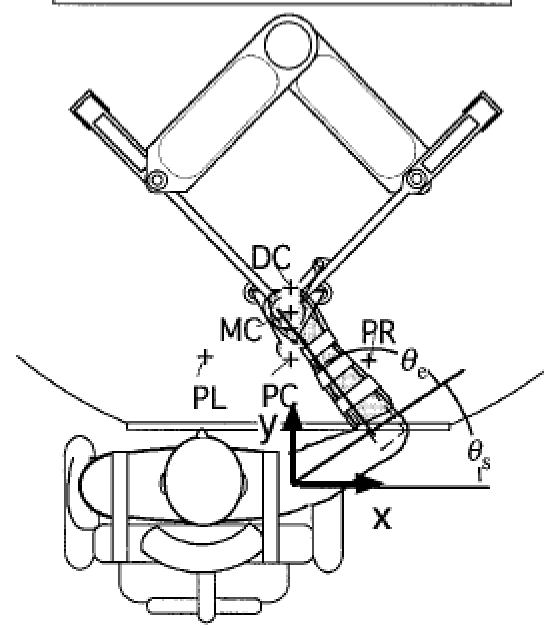
EMG and Stiffness

- Surface EMGs are highly correlated with static stiffness (Osu and Gomi 1999)
- EMGs are widely considered as the best candidate source for stiffness estimation of the human joints
  (Darainy et al 2004; Flash and Mussa Ivaldi 1990; Franklin et al 2003; Mussa Ivaldi et al 1985; Geribble et al 2003; Osu and Gomi 1999; Tsuji et al 1995 and Shin et al 2008)
- Few applications to robotics

EMG and Stiffness

Previous work on joint torque and stiffness estimation from EMG signals demonstrated feasibility, and indicated methods





“Estimation of MultiJoint Limb Stiffness from EMG Duirng Reaching Movements”,
D.W. Franklin,F. Leung, M. Kawato, T. E. Milner 2003

“Multijoint Muscle Regulation Mechanisms Examined by Measured Human Arm Stiffness and EMG Signals”
R. Osu and H. Gomi, J. Neurophysiol., 2005

EMG and Stiffness

- Muscle tensions correspond to the rectified EMGs of the arm muscles (Basnajian and De Luca 1984)
- The relationship between EMGs of agonistic and antagonistic muscles to the generated torques and stiffness in joints can be assumed linear in isometric conditions (Osu, Gomi Kawato *et al.* 2001). Neglecting cross-joint stiffness w.r.t. homologous joint stiffness (Gomi and Osu 1998) →

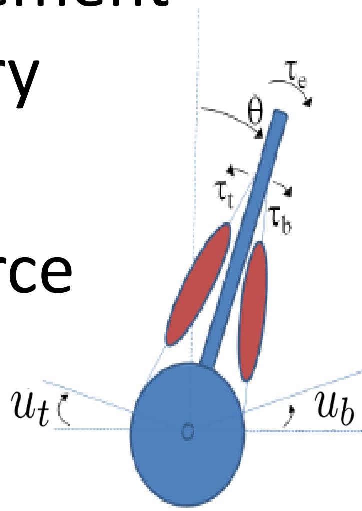
$$\tau = T(q) f, \quad \tau, q \in R^3, f \in R^6$$

$$\sigma = S(q) f, \quad \sigma = \left(\frac{\partial \tau_i}{\partial q_i} \right) \in R^3$$

EMG and Position Control

- EMG related to muscle force – not to displacement - indeed, a person can relax muscles at arbitrary positions (Wachholder)
- How to control slave position from master force information?
  - Control the robot force (with dynamic scaling)
  - Integrate the human arm dynamics to find position
  - Forget it

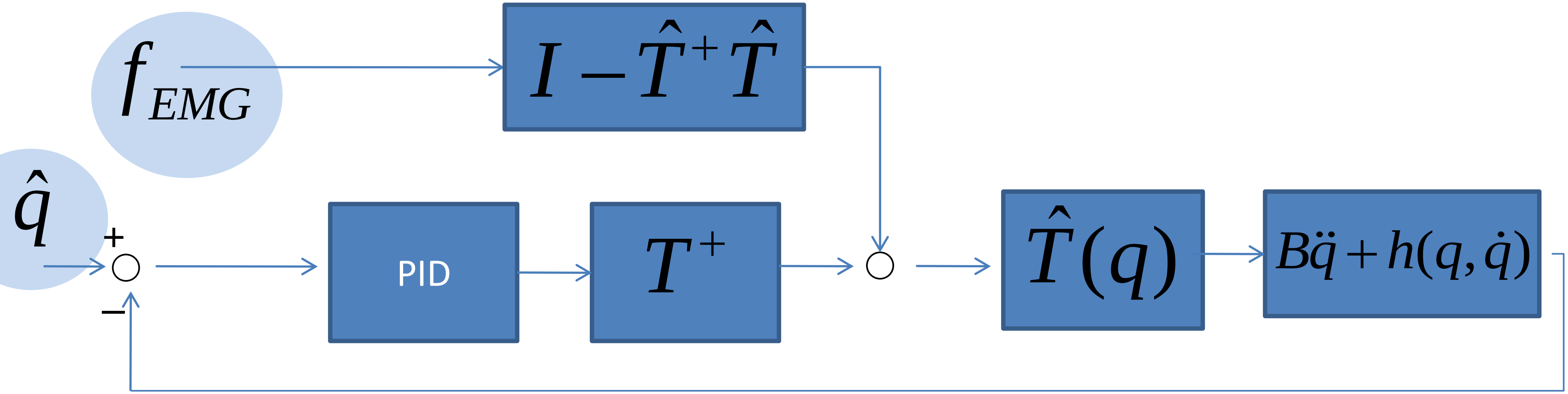
    There exist very cheap and accurate endpoint position sensors



Decoupled Position and Impedance Remote Control

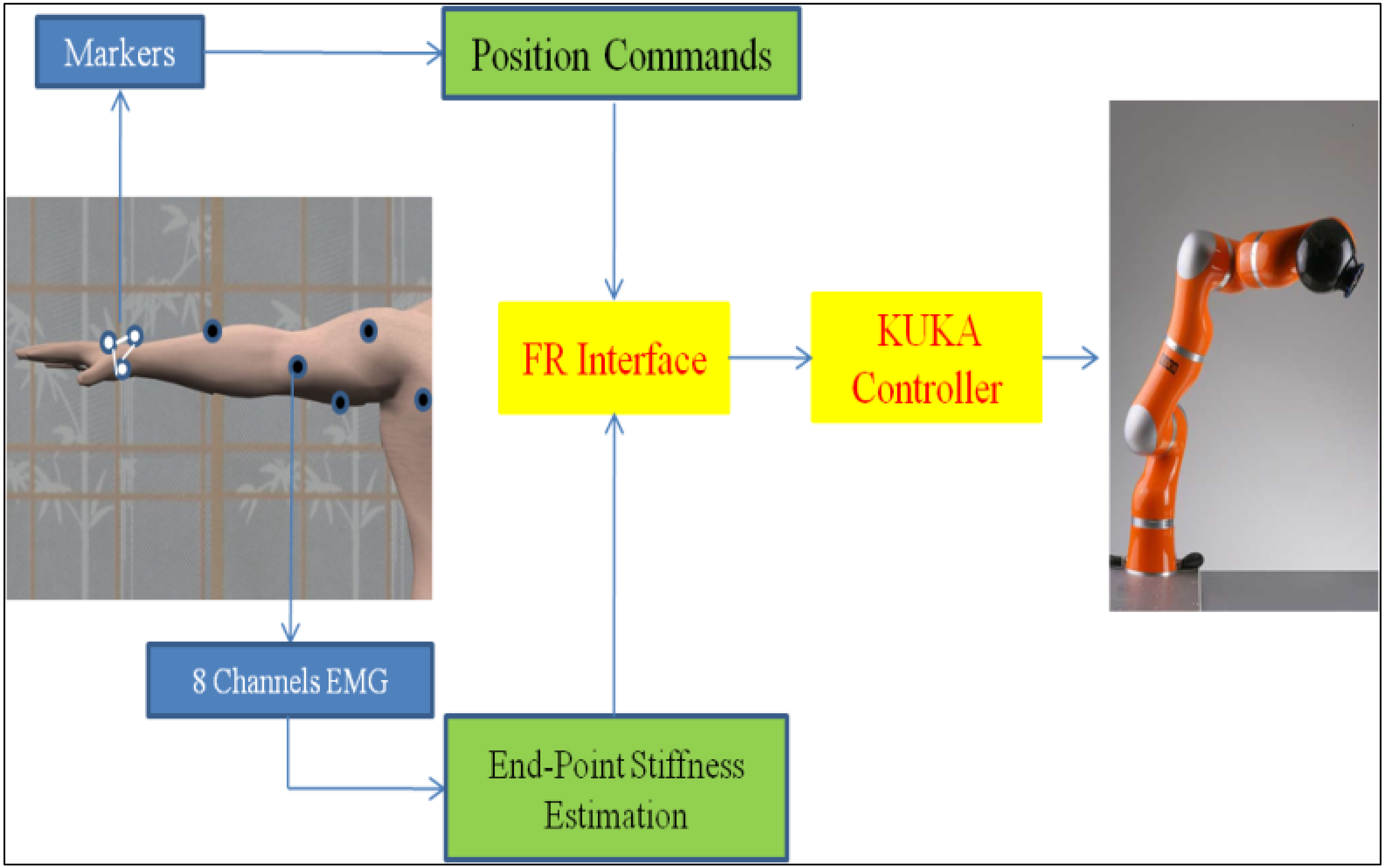
- Control is decoupled so that the error-prone EMG impedance measurements will not spoil the accurate position tracking control
- True only as far as there are no external forces: if there are, the control of stiffness will have implications on the position tracking error (by definition)

Decoupled Position/Stiffness Control of Robot Arms

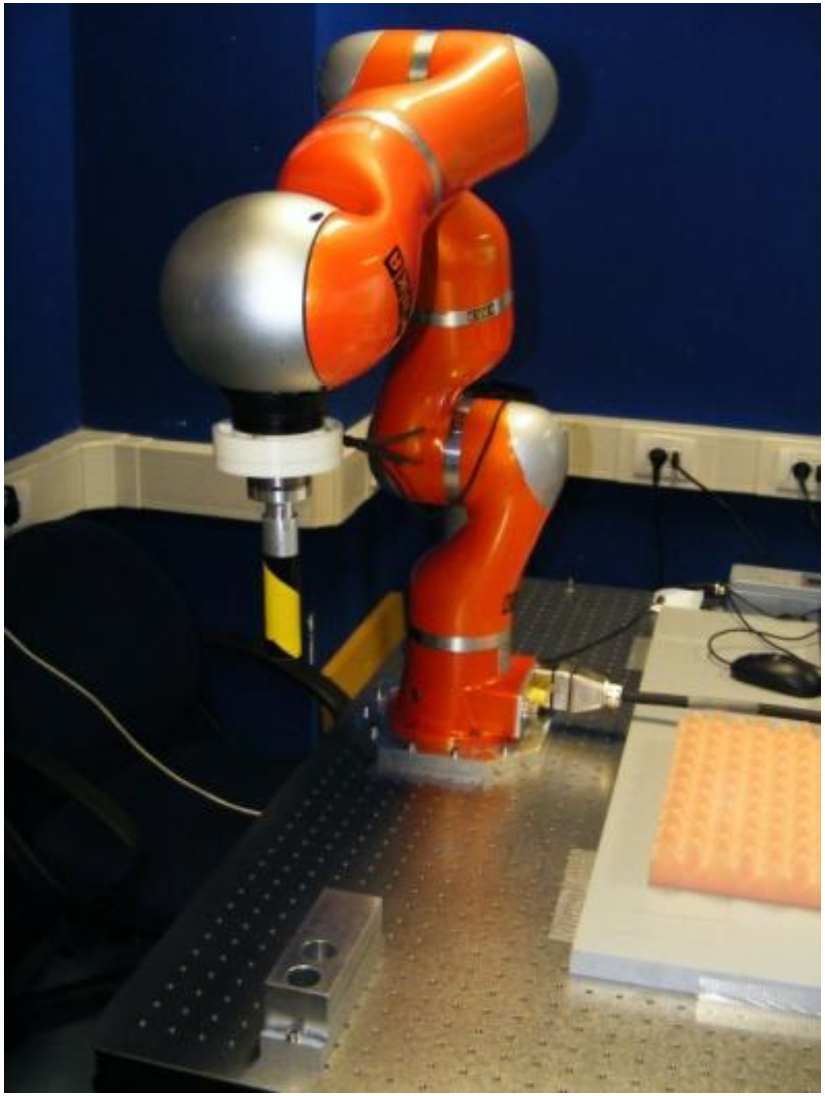


$$T^{+} = T^{T}(TT^{T})^{-1}$$

DPIRC Control Scheme



Experimental Setup



- Kuka LW Arm
- FR interface (Schreiber *et al* 2011)
- Advanced end-point Impedance Controller

•3D Optitrack motion capture system

Stiffness from EMG

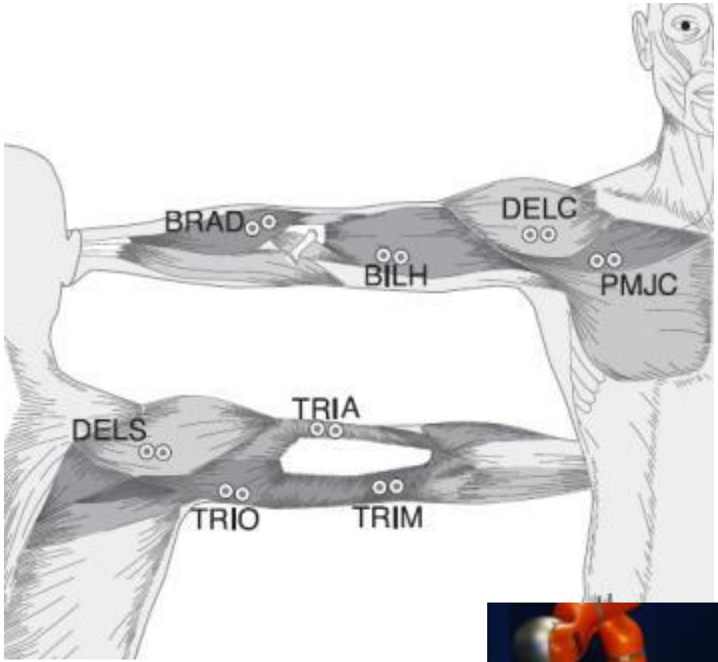
$$S_s = \sum_{i=1}^{n} \alpha_i . f(EMG_{ago-i}) - \sum_{j=1}^{n} \beta_j . f(EMG_{anta-j})$$

$$S_e = \sum_{i=1}^{n} \delta_i . f(EMG_{ago-i}) - \sum_{j=1}^{n} \mu_j . f(EMG_{anta-j})$$

s : shoulder   e : elbow

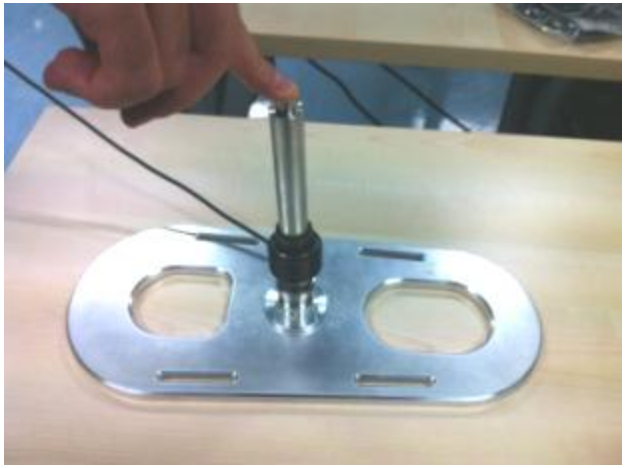
$fEMG$: filtered EMG

α, β, δ and μ : constant coefficients to be estimated

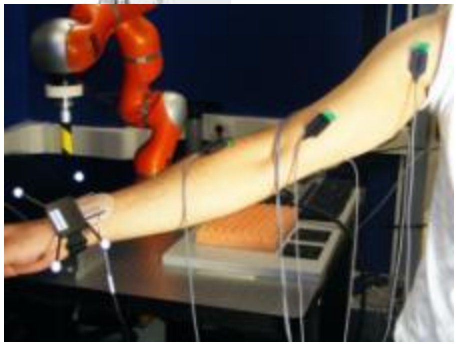


Electrode Placement

Calibration Experiments

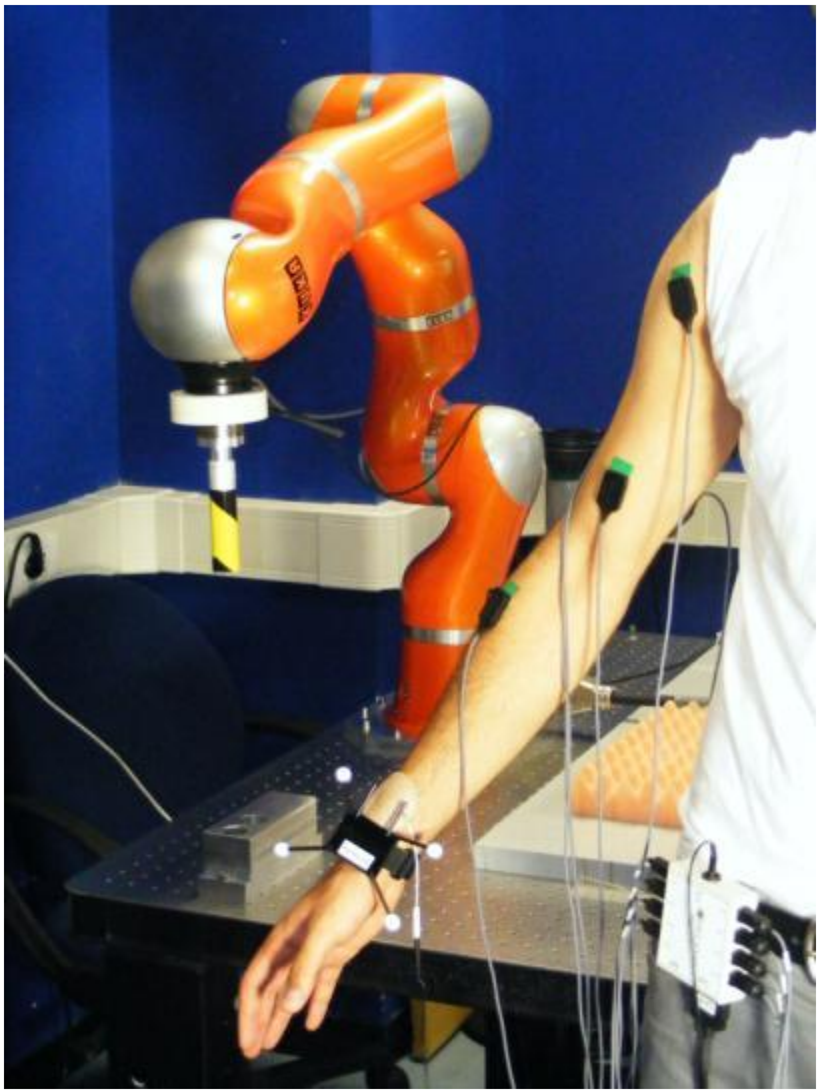


Instrumented Handle
with Spherical Joint



Isometric conditions
50 trials
Subjective stiffness
Coefficients calculated based on
Osu and Gomi (1999) ,
Osu, Gomi and Kawato et all (2001)).

Experiments



Disturbances

Interaction with soft, sticky surface

Position and Stiffness
Control
Sponge

Assembly Task

Position Control,
Constant Low Stiffness

Position Control,
Constant High Stiffness

Teleimpedance Control (1)

Teleimpedance Control (2)

Me

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Measuring Variable Impedance

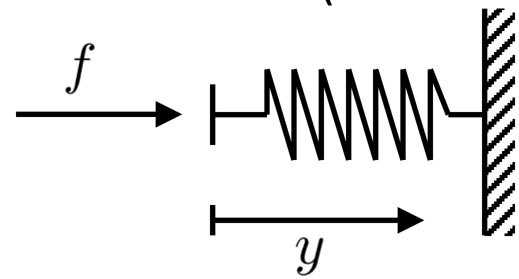
Giorgio Grioli[*], Alessandro Serio[+],
Nikos Tsagarakis[+] Irene Sardellitti[+]
Antonio Bicchi[*+]

[*]Centro "E. Piaggio", Università di Pisa
[^]IIT - Istituto Italiano di Tecnologia

Impedance for Non-Linear Mechanical Systems

- Simplest notion of mechanical impedance: Linear stiffness (Hooke's Law)

$$f = Ky$$



- Generalization to **Non-Linear Springs**:
  - Partial derivative

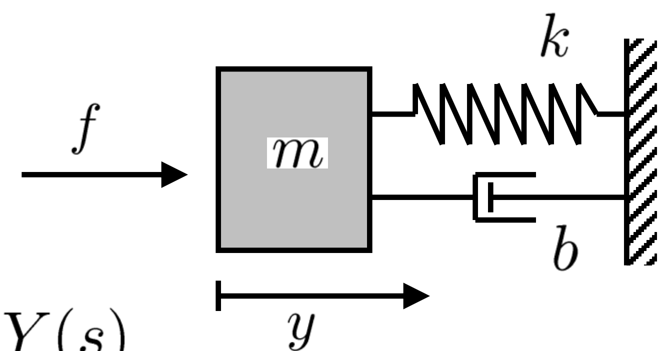
$$y = f(y) \quad \Rightarrow \quad \sigma(y) = \frac{\partial f(y)}{\partial y}$$

- Generalization to **Dynamic Systems**:
  - Laplace Transform: **Impedance**

$$f = m\ddot{y} + b\dot{y}\ ky$$

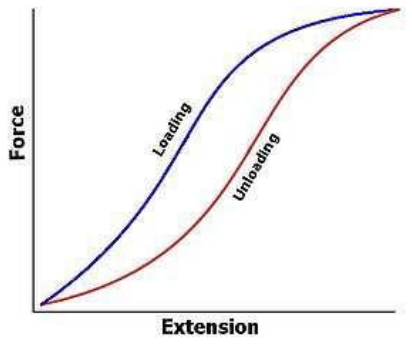
$$\Downarrow$$

$$F(s) = (ms^2y + bs + k)Y(s) = Z(s)Y(s)$$



i.e., an *operator* that sends functions (and initial conditions) in functions

Impedance for Non-Linear Mechanical Systems



- Generalizing Impedance:
  - Graph $G \subset F \times Y \times DY \times D^2Y \times U$
    - Analytical Description

$$G(f, y, \dot{y}, \ddot{y}, u) = 0$$

  - At a Regular point d_0:

    Locally there exists an implicit function

$$k(d) = -\left(\frac{\partial G(d)}{\partial f}\right)^{-1} \frac{\partial G(d)}{\partial y}$$

$$b(d) = -\left(\frac{\partial G(d)}{\partial f}\right)^{-1} \frac{\partial G(d)}{\partial \dot{y}}$$

$$m(d) = -\left(\frac{\partial G(d)}{\partial f}\right)^{-1} \frac{\partial G(d)}{\partial \ddot{y}}$$

    - $f(y, \dot{y}, \ddot{y}, u)$ ⇒

    Fréchet differential

$$\delta f = m(d)\,\delta\ddot{y} + b(d)\,\delta\dot{y} + k(d)\,\delta y + \nu(d)\,\delta u$$

Admittance View

$$\ddot{y} = g(y, \dot{y}, f, u)$$

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ g(x_1, x_2, f, u) \end{bmatrix}$$

$$\dot{\tilde{x}} = \Gamma(t)\tilde{x} + \Theta(t)\tilde{f},$$

where

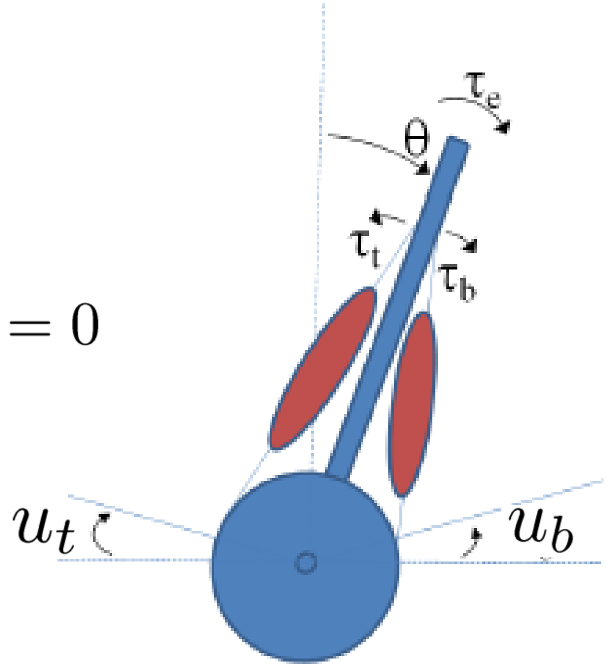
$$\Gamma(t) = \begin{bmatrix} 0 & 1 \\ -\frac{k(d)}{m(d)} & -\frac{b(d)}{m(d)} \end{bmatrix}, \quad \Theta(t) = \begin{bmatrix} 0 \\ \frac{1}{m(d)} \end{bmatrix}.$$

An example

- Antagonist "muscle" system
  - Dynamics:

$$I\ddot{\theta} + \beta\dot{\theta}|\dot{\theta}| - \tau_b + \tau_t - mgl\sin\theta - \tau_e = 0$$

where
$$\tau_b = -\alpha(\theta_b - u_b)^2$$
$$\tau_t = -\alpha(\theta_t - u_t)^2$$



  - Gen. Stiffness: $k(\theta) = 2\alpha(\pi - u_b - u_t) - mgl\cos(\theta)$
  - Gen. Damping: $b(\dot{\theta}) = 2\beta|\dot{\theta}|$

An example

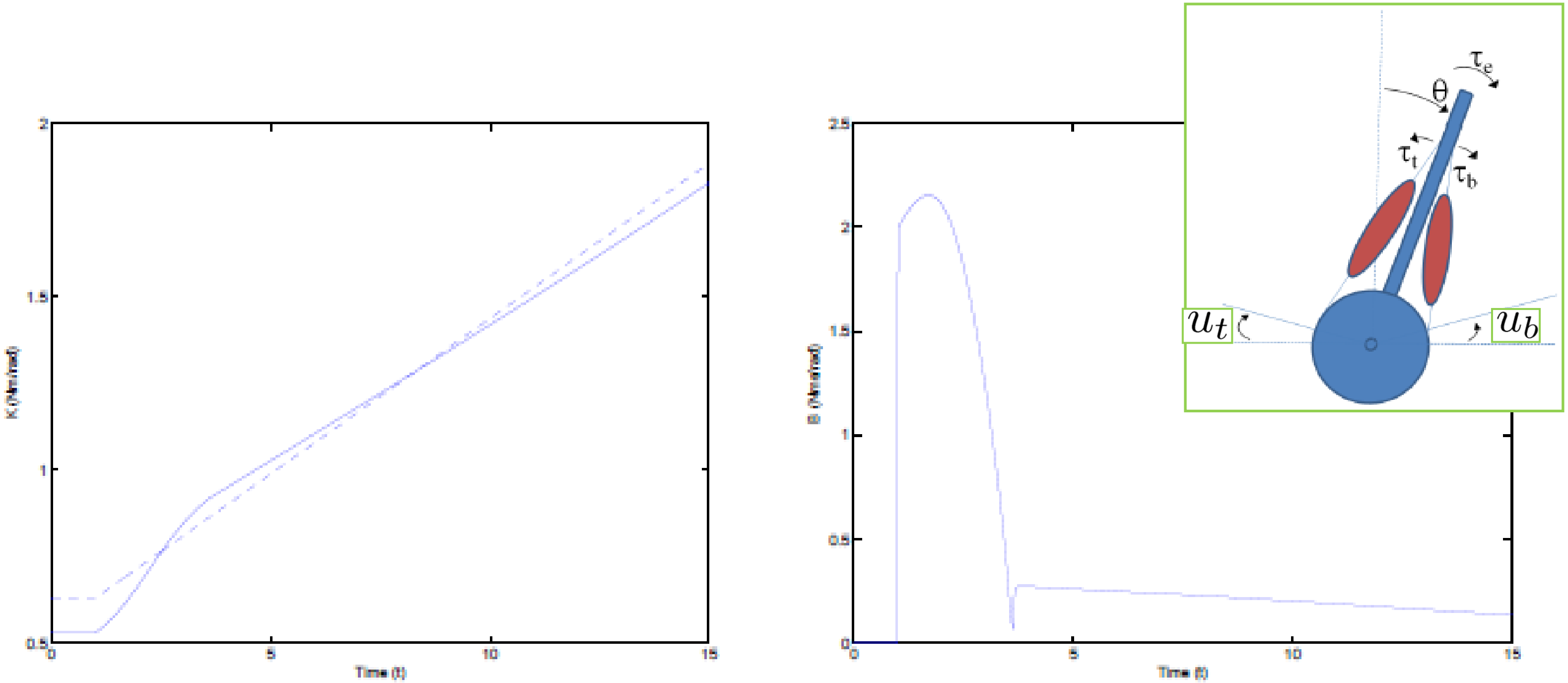


Fig. 4. Generalized stiffness (left - dashed is without gravity term) and generalized damping for the example with acuators as in (4), subject to a unit step in external torque at $T = 1$s, and with time-varying reference angle $\lambda_b(t) = \lambda_t(t)$ linearly decreasing from $\pi/3$ at $T = 1$s to 0 at $T = 15$s. Numerical values used in simulation as in fig. 3, except for $\alpha = 0.3$.

Measuring Impedance

- Measurements are at the basis of science

> *"Misura ciò che e misurabile, e rendi misurabile ciò che non lo è"*
> *(Measure what is measurable, and make measurable what is not)*
>
> **Galileo**

- Feedback needs measuring
  - Measuring impedance is needed for control of VIA actuators
- Impedance is a differential operator not a *physical quantity* in a strict sense

> "**Physical Quantity***:* a property of a phenomenon, body, or substance, where the property has a magnitude that can be expressed as a number and a reference"
>
> **International Vocabulary of Metrology (VIM).**
> **Basic and General Concepts and Associated Terms.**

Measuring Impedance

Impedance Measurements State of the Art

- In ME
- In Biomechanics
- In Robotics, etc.

Common Characteristics

- **Typically:** repeated experiments with probing perturbations
- **Mostly:** not applicable in real time
- **Always*:** linear, time invariant impedance

Measuring Linear Impedance

- Simple case $f = m\ddot{y} + b\dot{y} + ky \qquad m,\, b,\, k > 0$
  - Build a non-linear equivalent system
  - Observability Co-distribution

$$\begin{cases} \dot{z} = \begin{bmatrix} z_2 \\ z_1 z_3 + z_2 z_4 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ z_5 \\ 0 \\ 0 \\ 0 \end{bmatrix} f \\ y = h(z) = z_1 \end{cases} , \; z = \begin{bmatrix} y \\ \dot{y} \\ k \\ b \\ m \end{bmatrix}$$

$$\Omega(z) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ z_3 & z_4 & z_1 & z_2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ z_3 z_4 & z_3 + z_4^2 & z_2 + z_1 z_4 & z_1 z_3 + 2 z_2 z_4 & 0 \\ 0 & 0 & 0 & z_5 & 4 \end{bmatrix}$$

$$y, \dot{y} \neq 0$$

⇓

$$\text{rank}(\Omega) = 5$$

⇓

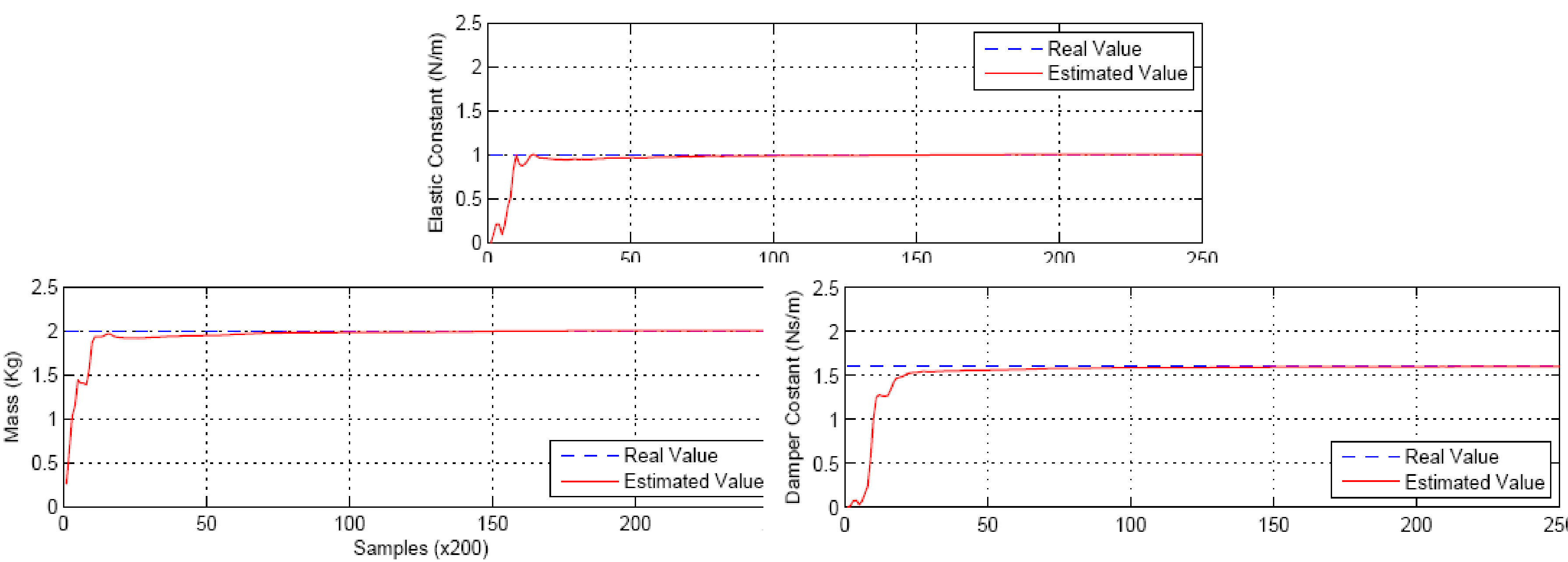
OBSERVABLE!

Measuring Linear Impedance

$$f = m\ddot{y} + b\dot{y} + ky$$

Build a regression, or non-linear observer

e.g. an Extended Kalman Filter



Measuring Nonlinear Impedance

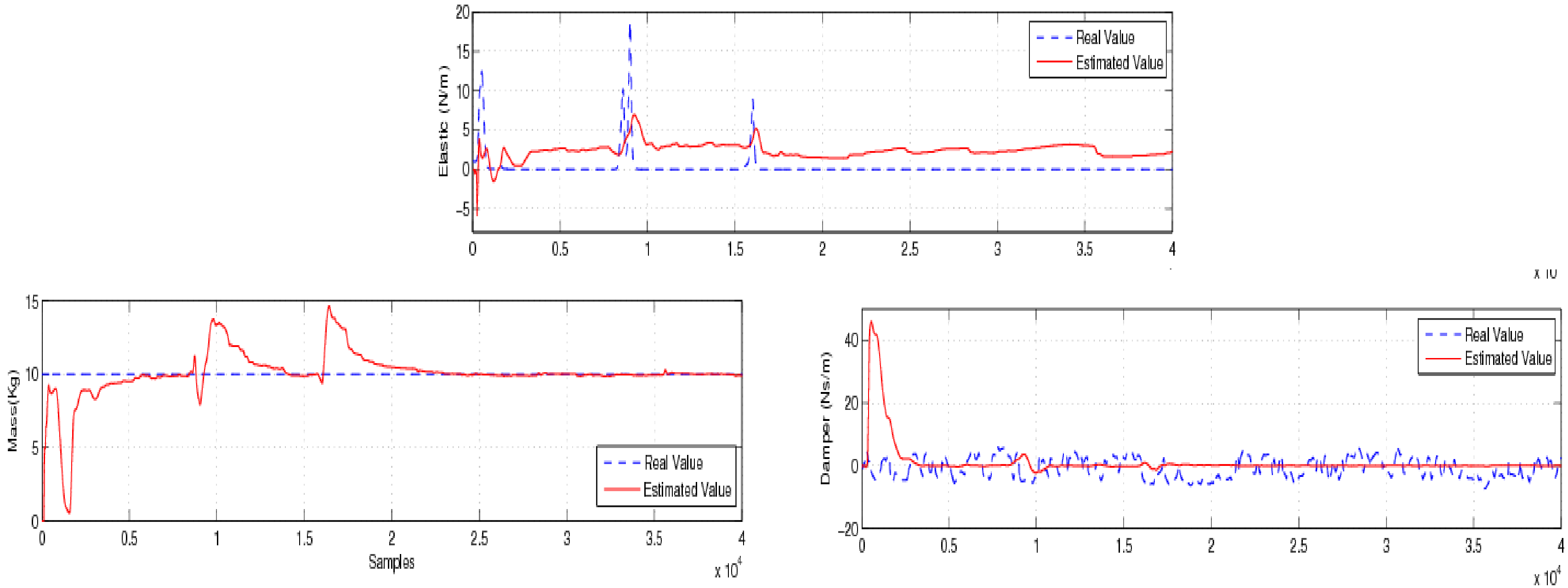
$$f = m(\ddot{y}) + b(\dot{y}) + k(y)$$

The same approach is no longer possible (at least, not trivially)

$$\begin{cases} \dot{z} &= \begin{bmatrix} z_2 \\ z_1 z_3 + z_2 z_4 \\ ? \\ ? \\ ? \end{bmatrix} + \begin{bmatrix} 0 \\ z_5 \\ ? \\ ? \\ ? \end{bmatrix} f \\ y &= h(z) = z_1 \end{cases}$$

Measuring Nonlinear Impedance

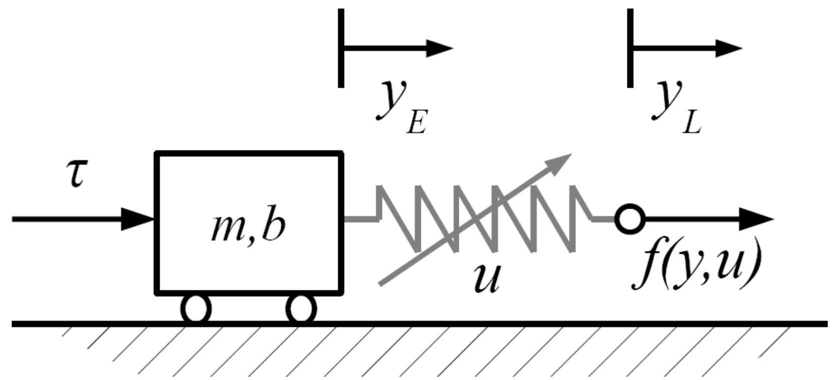
Using EKF with a nonlinear impedance…



Variable Stiffness Observers

– given

$$\tau = m\ddot{y} + b\dot{y} + f\left(y, u\right)$$



– differentiation yields

$$\dot{\tau} = my^{(3)} + b\ddot{y} + \sigma\dot{y}$$

$$\dot{\tilde{\tau}} = \dot{\tau} - \dot{\hat{\tau}} = [\ldots] = \tilde{\sigma}\dot{y}$$

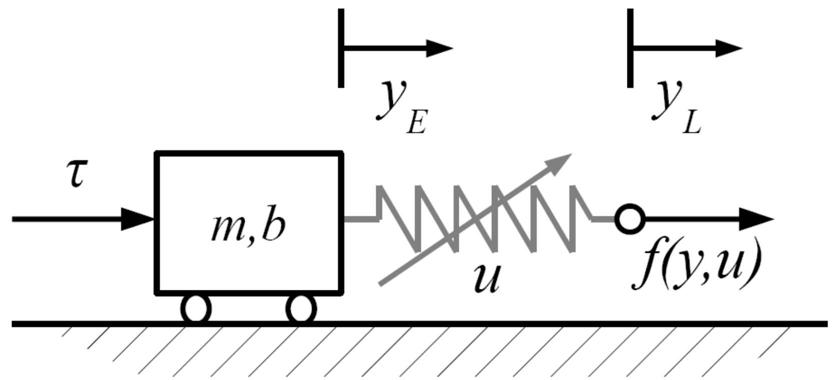
– build an estimate

$$\dot{\hat{\tau}} = my^{(3)} + b\ddot{y} + \hat{\sigma}\dot{y}$$

The Variable Stiffness Observer

Th.: The update law

$$\dot{\hat{\sigma}} = \alpha \dot{\tilde{\tau}} \operatorname{sgn}(\dot{y})$$



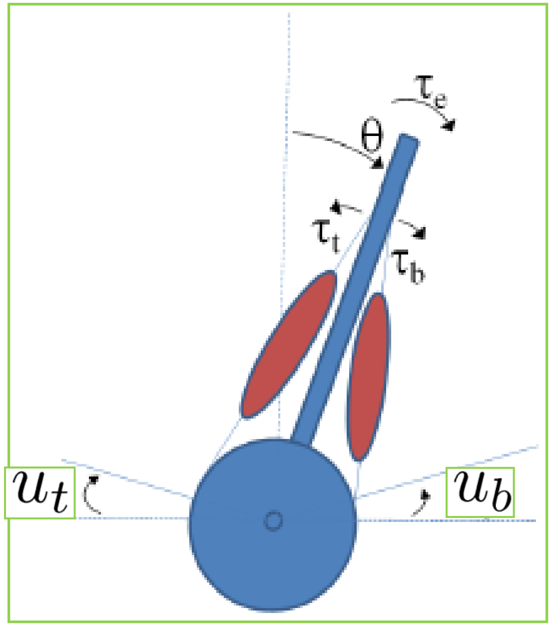
converges to within an **Uniformly Ultimately Bounded** error region around the **real stiffness value**

$$|\tilde{\sigma}| > \frac{|\sigma_y|}{\alpha} + \left(|f_u| + \frac{|\sigma_u|}{\alpha} \right) \frac{\dot{u}}{\dot{y}}$$

"A Non-Invasive, Real-Time Method for Measuring Variable Stiffness"
G. Grioli, A. Bicchi
Robotic Science and Systems 2010, Zaragoza, Spain. Submitted paper

The Variable Stiffness Observer

$$|\tilde{\sigma}| > \frac{|\sigma_y|}{\alpha} + \left(|f_u| + \frac{|\sigma_u|}{\alpha} \right) \frac{\dot{u}}{\dot{y}}$$

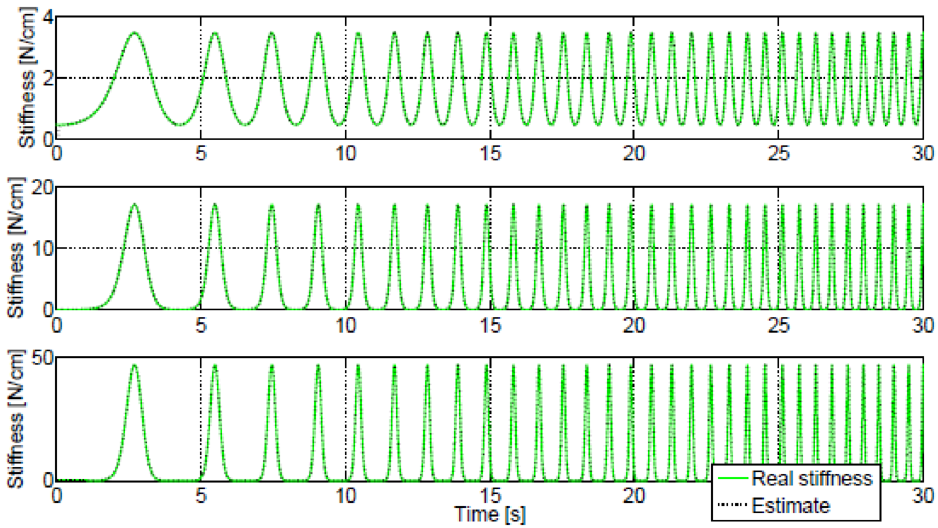


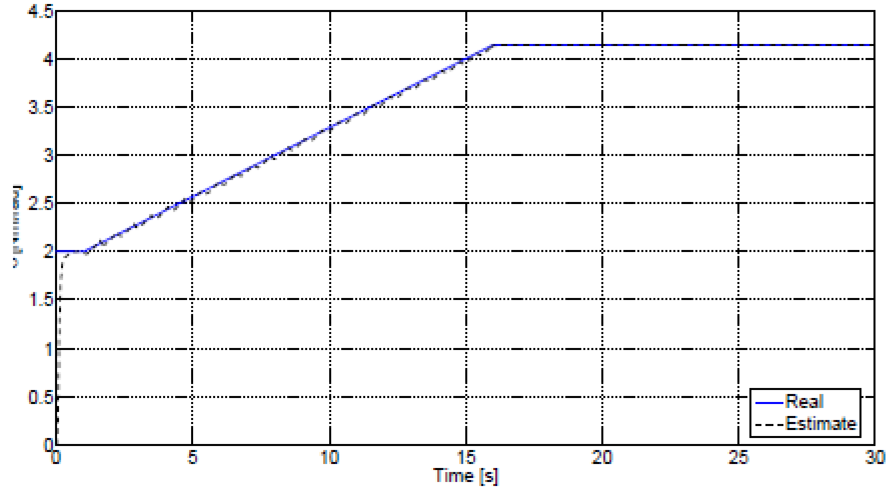
•The steeper stiffness changes with position and input, the larger is the error

•Large co-contraction velocity with slow limb displacement may cause large errors

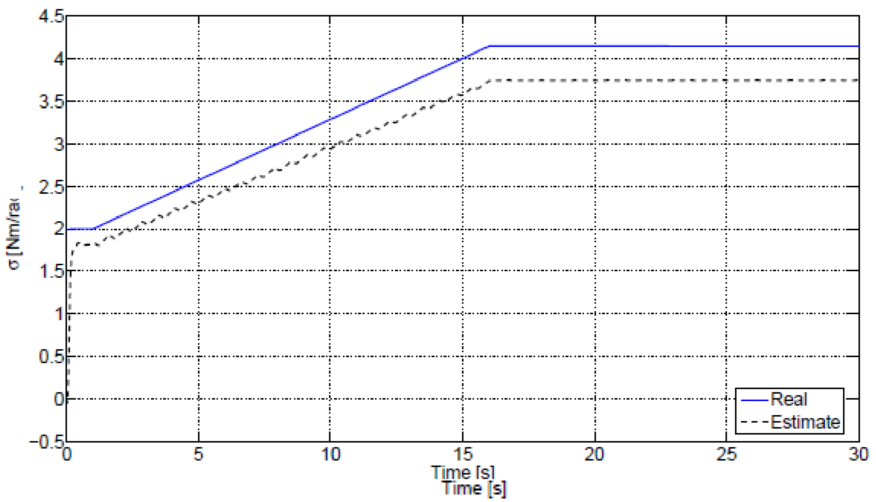
VSO - Simulations

(a) tracking





(a) example 1



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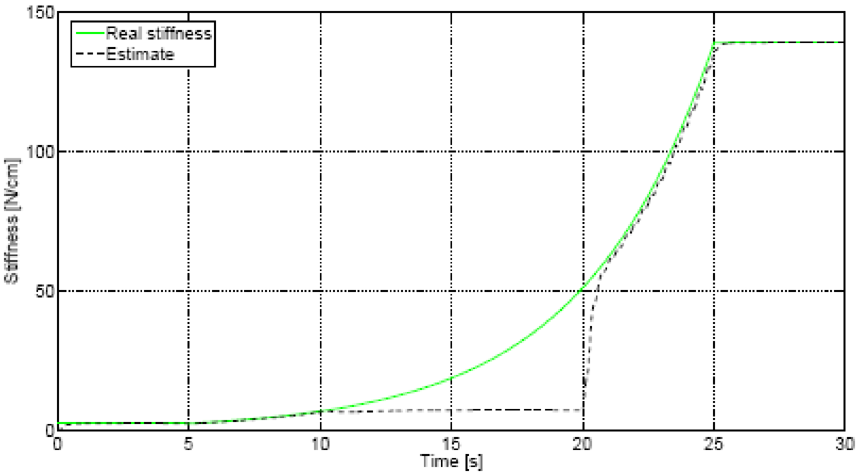
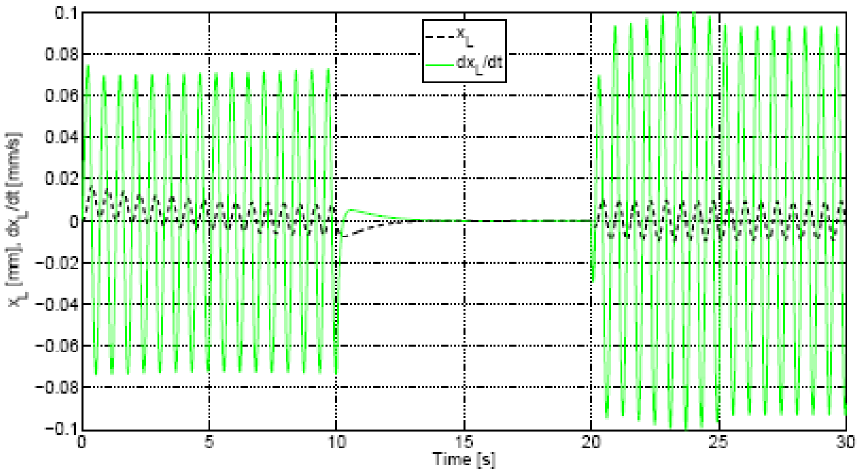
More interesting Simulations

– When the limb stops…

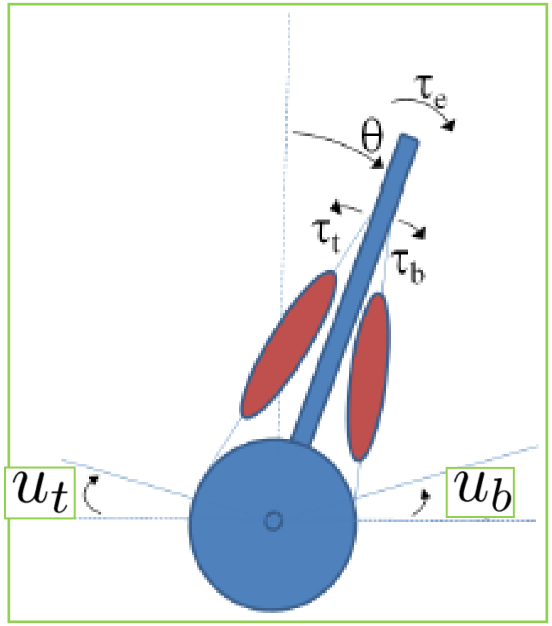


– Errors in m,b



The Variable Stiffness Observer

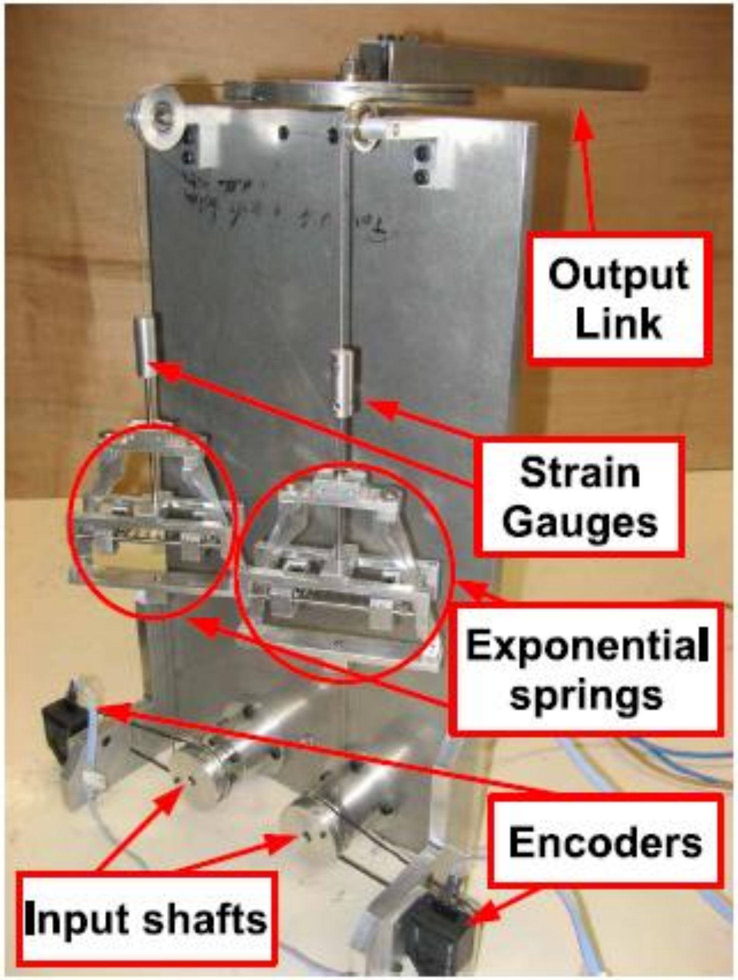
$$|\tilde{\sigma}| > \frac{|\sigma_y|}{\alpha} + \left(|f_u| + \frac{|\sigma_u|}{\alpha} \right) \frac{\dot{u}}{\dot{y}}$$



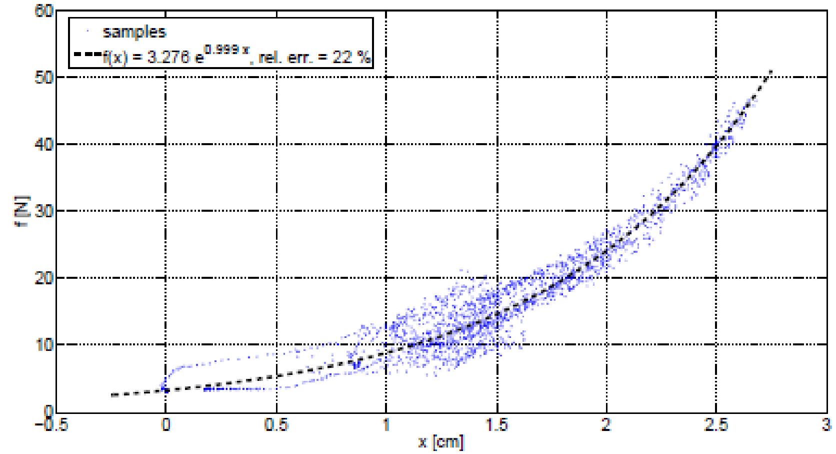
•The steeper stiffness changes with position and input, the larger is the error

•Large co-contraction velocity with slow limb displacement may cause large errors

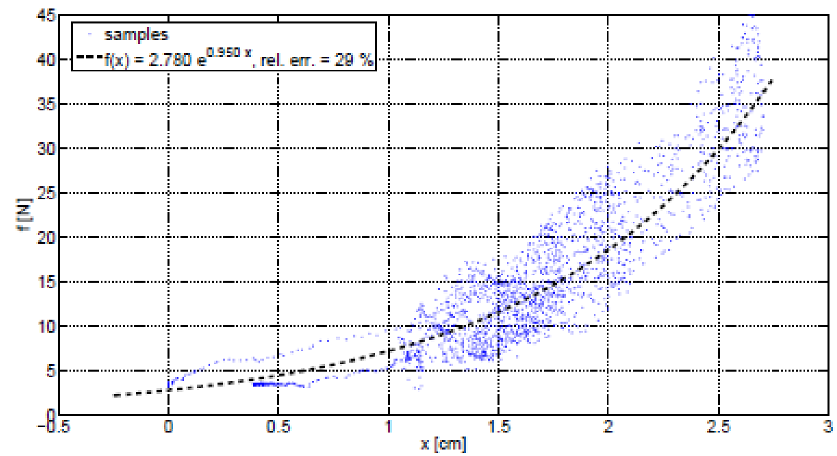
Experimental Results



(a) left spring



(b) right spring



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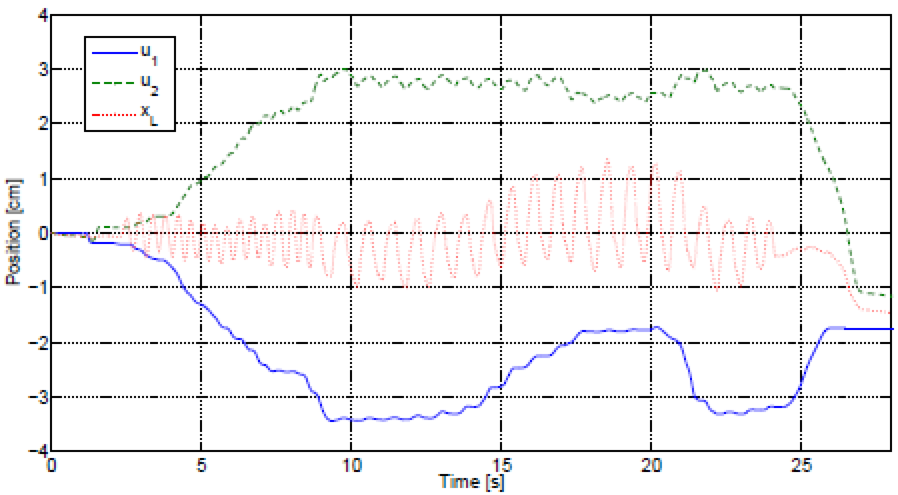
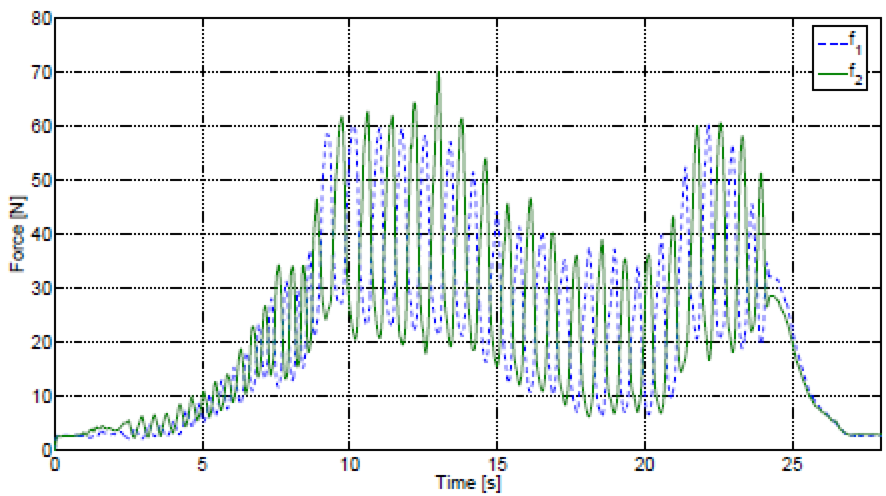
Spring Calibration

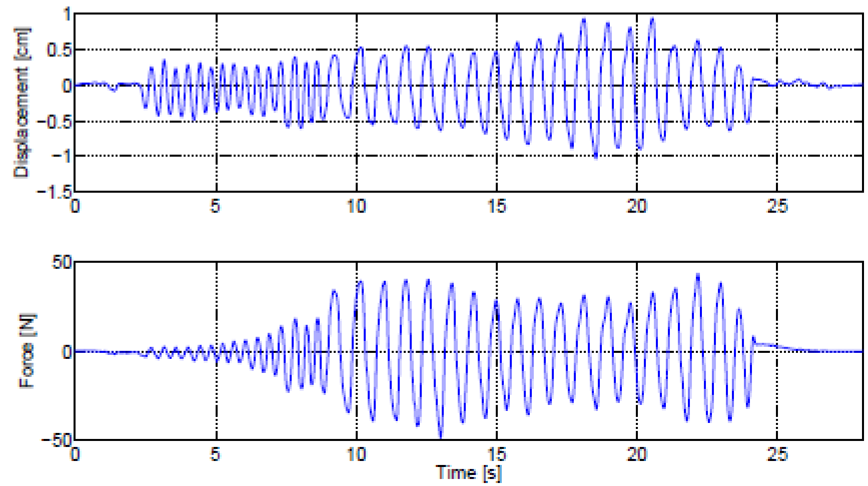
Experimental Results

(a) positions



(b) forces

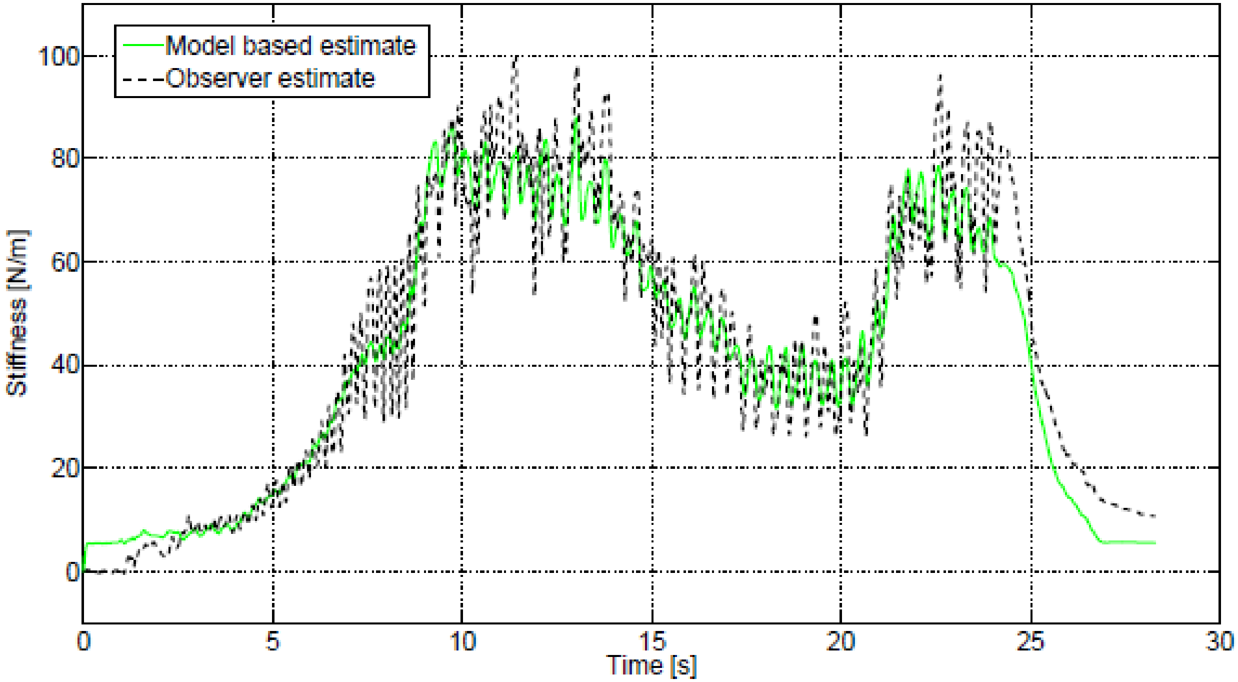




Position and Force

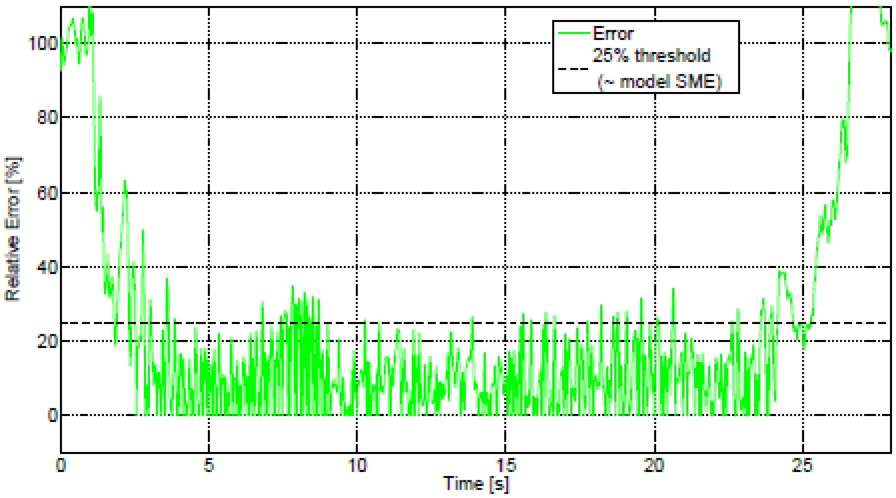
Raw data

Experimental Results



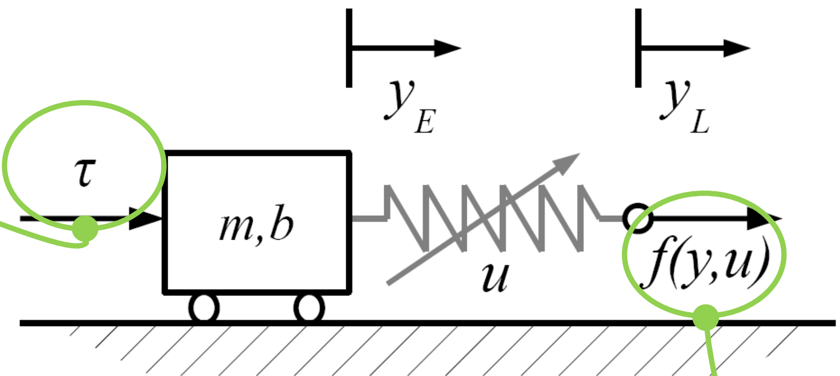
Observed Stiffness

Relative Error



VSO – Mass and Damping

Can we observe stiffness without knowing ***m*** and ***b*** ?



NO, if we measure only the applied torque

– e.g. human measurements

YES if we measure the elastic force ("inside" the joint)

– e.g. robots

Impedance Observers

Non-parametric Stiffness observer

Evolutions:

- **Parametric Stiffness observer**
  (ICRA2011)

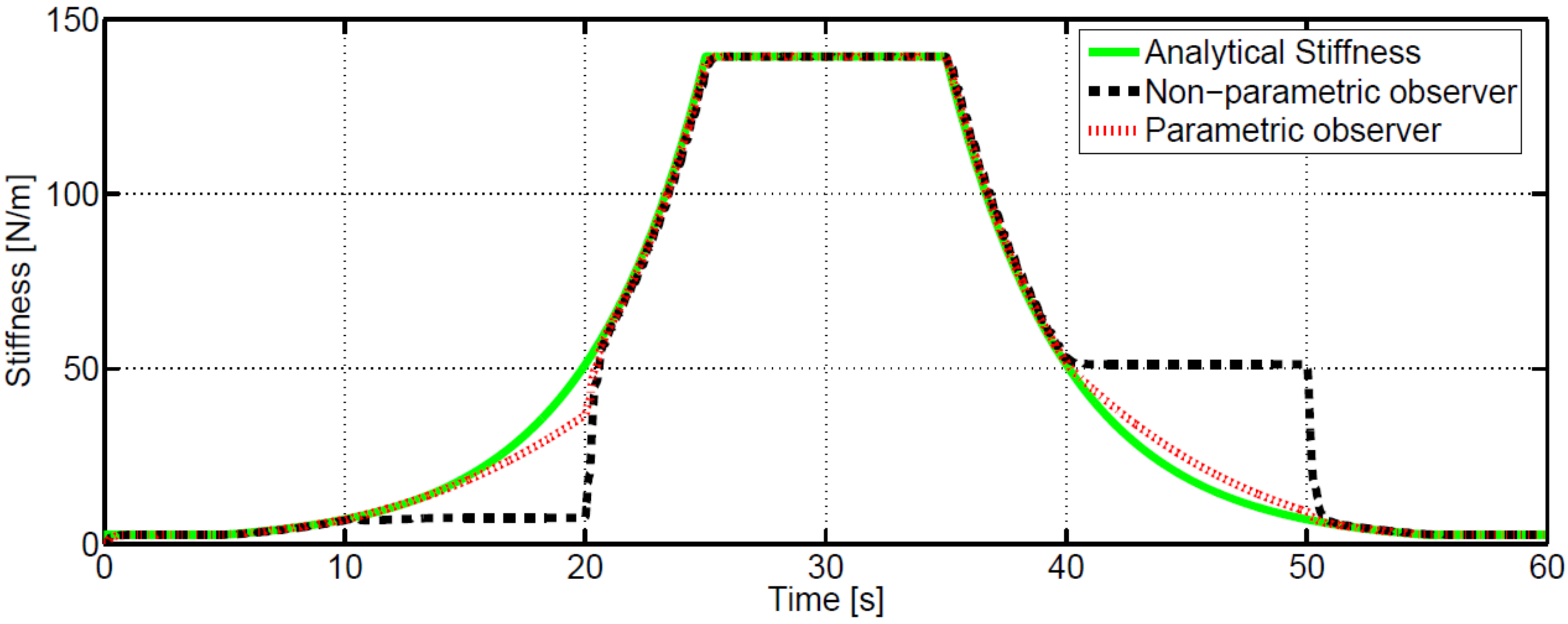
- **Decoupled Impedance Observer**
  (ICRA2011)

Impedance Observers

Parametric Stiffness observer

Fills in the non-observable gaps (if internal measurements are available)

(a) stiffness tracking



Impedance Observers

Parametric Stiffness observer

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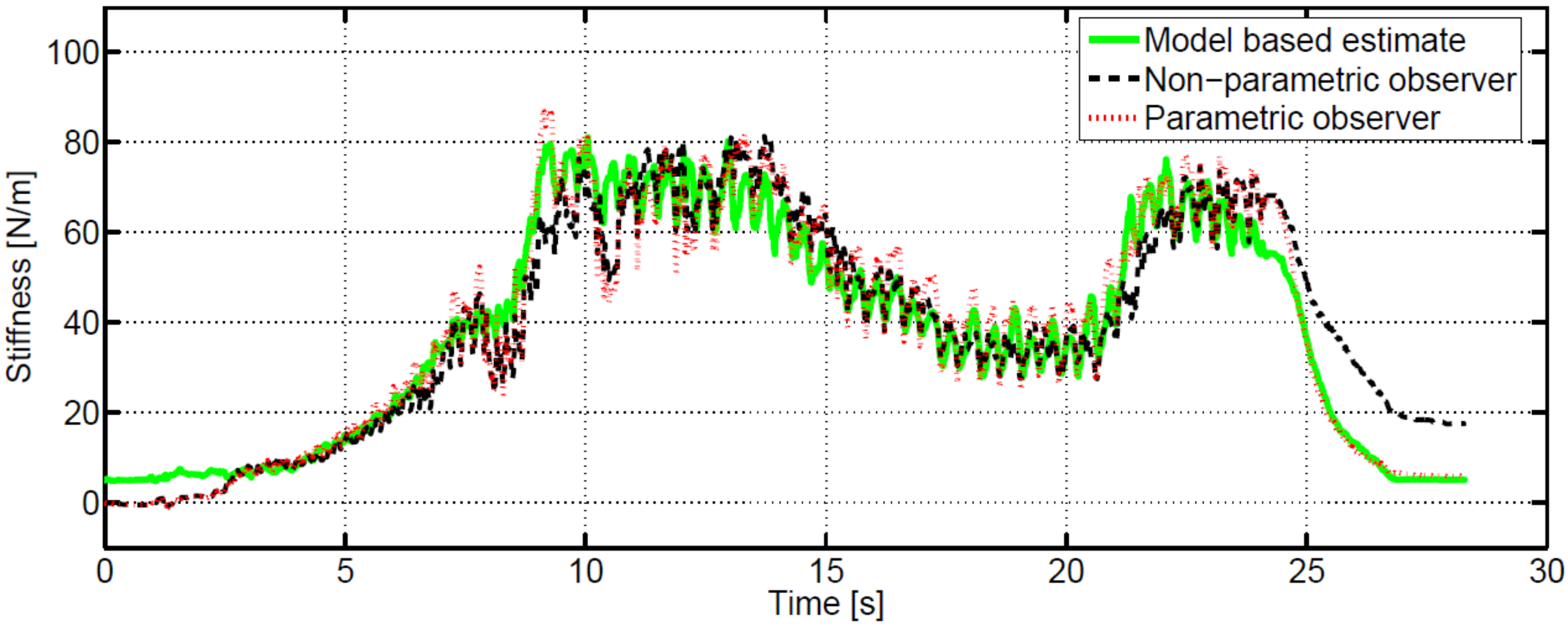
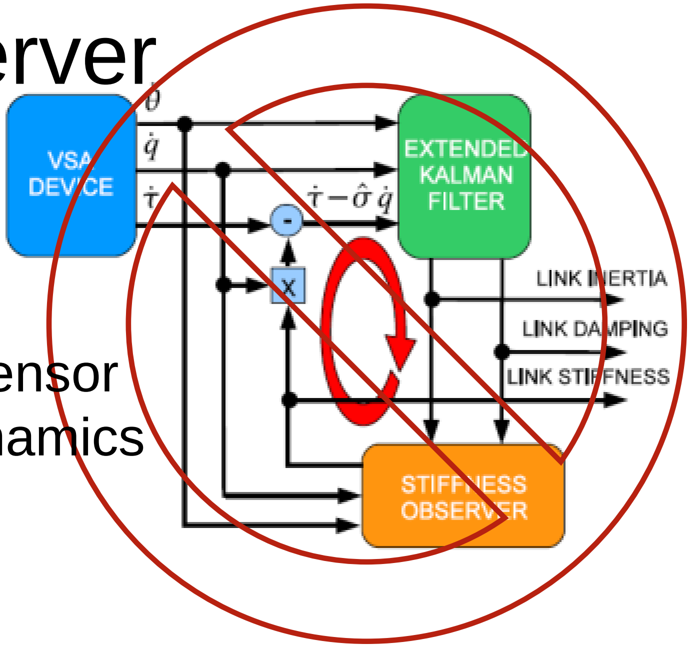


Impedance Observers

Decoupled Impedance observer

– Idea:
  - Integrate stiffness observer with EKF for linear impedance
  - Exploit clever placement of torque sensor to avoid interaction of estimation dynamics
– Cons:
  - Not applicable to all kind of VSAs
  - Requires some knowledge of the motor model
– Pros:
  - **Reconstructs the whole impedance on the link side**

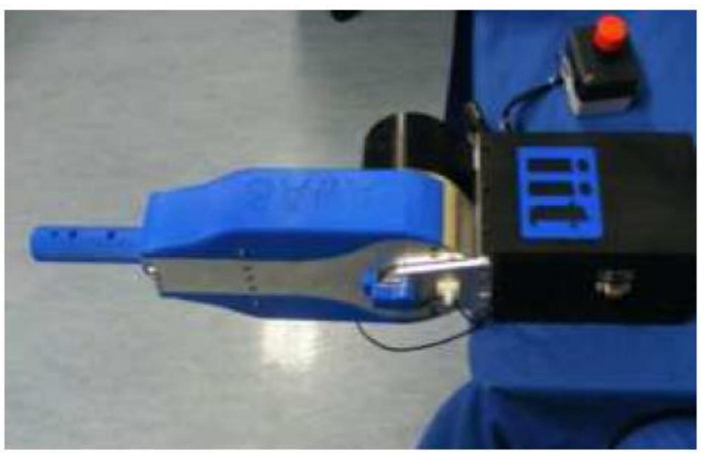


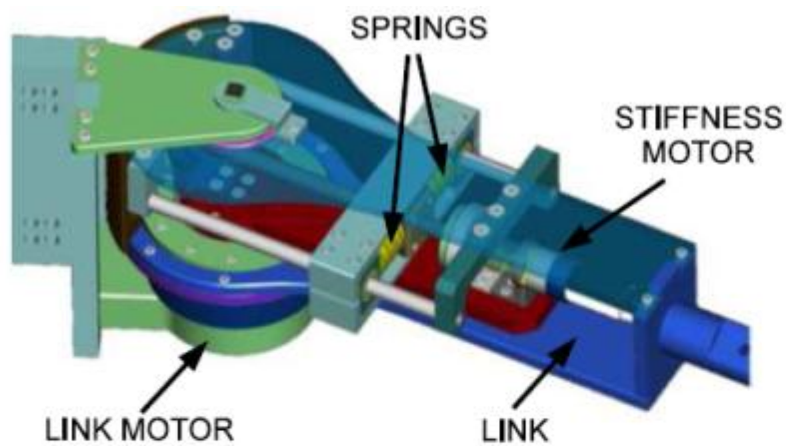
Impedance Observers

Decoupled Impedance observer

– **Reconstructs the whole impedance (K,D,I)**

Successfully applied to AwAS joint in Genoa



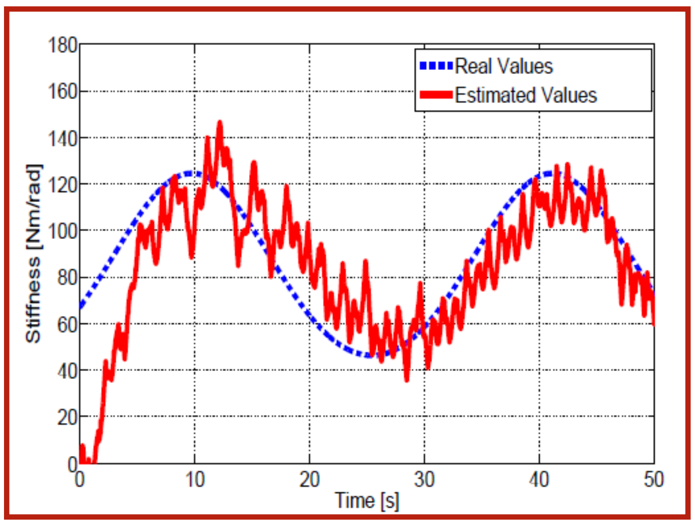


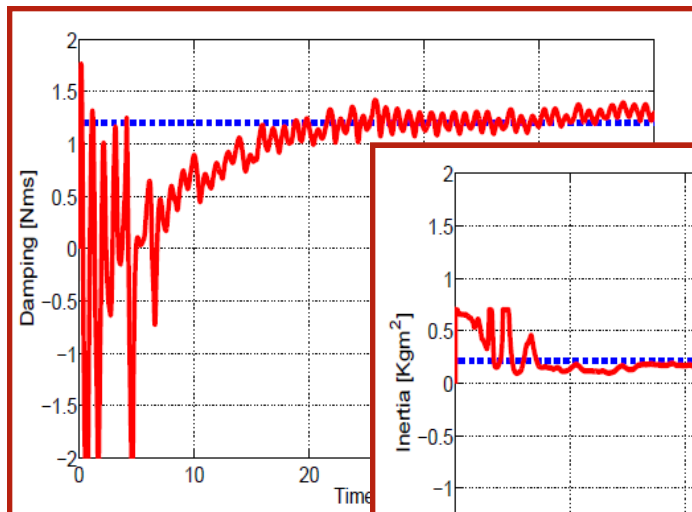
Impedance Observers

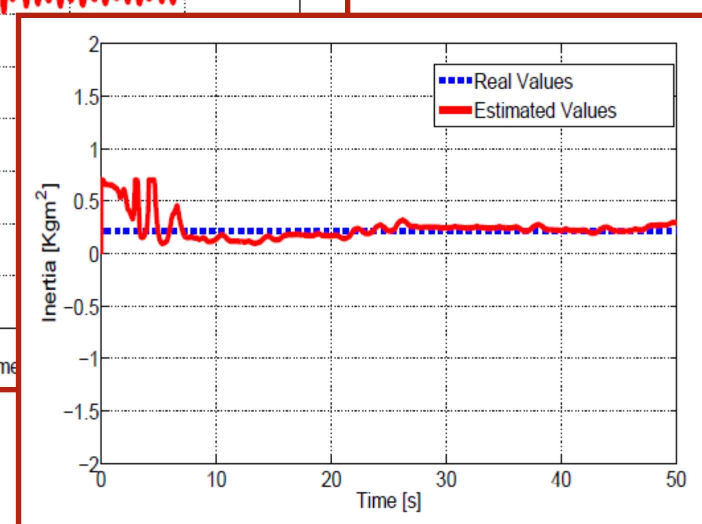
Decoupled Impedance observer

– **Reconstructs the whole impedance (K,D,I)**

Successfully applied to AwAS joint in Genoa







Summary - VSO

- A discussion of nonlinear impedance definitions
- Real-time, non-invasive algorithms to estimate stiffness
- Use of "dirty" derivatives increases error, do not pose threats to filter stability
- Can be easily replaced by torque error, even integrated (residuals – see e.g. De Luca *et al.* 2011)
- Open issues
  - Extend to *n-dof's*
  - Observe (nl, tv) generalized mass and damping
  - Control impedance in closed loop
  - Apply beyond robotics

$$\dot{\tilde{\tau}} = \dot{\tau} - \dot{\hat{\tau}} = [\ldots] = \tilde{\sigma}\dot{y}$$

Outline

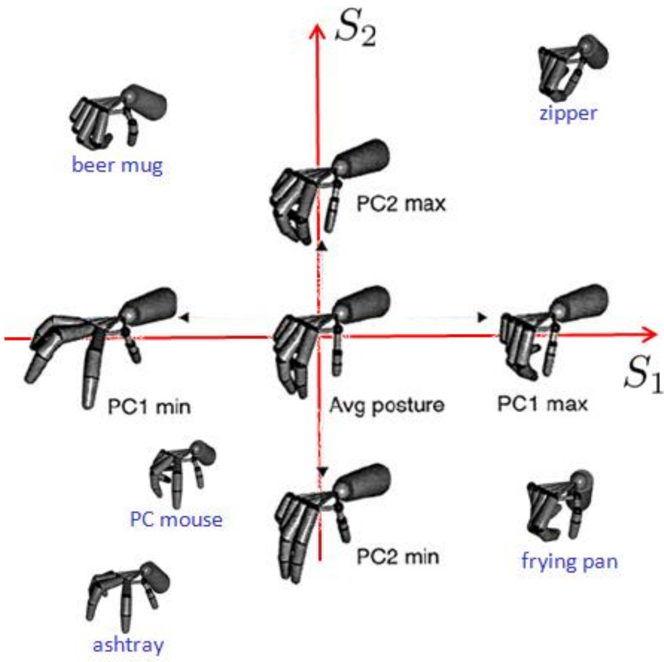
- A bit of a retrospective
- Using Variable Impedance
  - Optimal control
    - Safety oriented
    - Performance oriented
    - Energy oriented
  - Tele-Impedance
- Measuring Variable Impedance
- VSA and Hands
- Design

The Redundancy Problem: Synergies in the Hand Motor System

- Extensive neuroscientific evidence for the ***existence of sensorimotor synergies and constraints***
  Babinski (1914!), Bernstein, Bizzi, Arbib, Jeannerod, Wolpert, Flanagan, Soechting, Sperry, …

- Quantitative work on hand postural synergies dates back a decade only

Postural Synergies



Santello, Flanders, Soechting
J. Neuroscience, 1998

- Santello et al. (1998) investigated the hypothesis that *"learning to select appropriate grasps is applied to a series of inner representations of th complexity, which varies with experience and degree of accuracy required."*
  - 5 subjects were asked to shape their hands in order to mime grasps for a large set (57) of familiar objects;
  - Joint values were recorded with a CyberGlove;
  - Principal Components Analysis (PCA) of these data revealed that the first two Principal Components or postural synergies account for $\sim 84\%$ of the variance, first three $\sim 90\%$;
  - PCs (eigenvectors S_i of the Covariance Matrix) can be used to define a basis for a subspace of the joint space.

The Shape of Synergies

Postural ***synergies*** (aka **primitives**, ***eigengrasps*** or ***principal grasp components***) are the eigenvectors of the joint data covariance matrix;

First synergies contain most of hand posture information;
Higher-order synergies used for fine adjustments

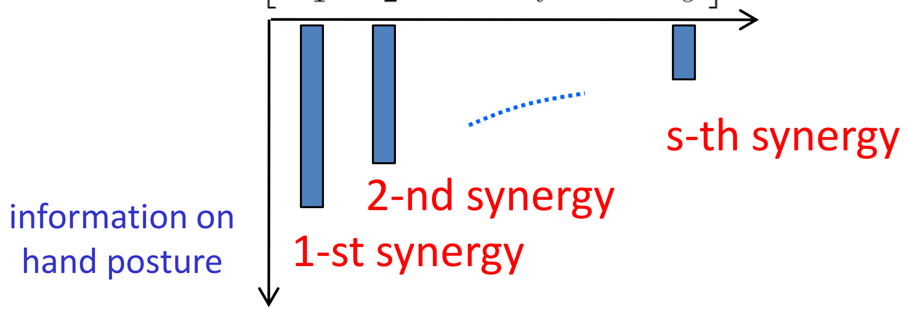
PCs can be collected in a synergy matrix:

$$S = \left[\begin{array}{c:c:c:c:c} \vdots & \vdots & \vdots & \vdots & \\ S_1 & S_2 & S_3 & \cdots & S_n \\ \vdots & \vdots & \vdots & \vdots & \end{array}\right]$$

S_1 (1-st synergy) S_2 (2-nd synergy) S_3 (3-rd synergy)

Model of a Hand with "*s*" synergies

$$\mathbf{S} = \left[\, S_1 \,\vdots\, S_2 \,\vdots\, \cdots \,\vdots\, S_i \,\vdots\, \cdots \,\vdots\, S_s \,\right]$$



Straightforward Kinematic interpretation:

- Joint configurations must belong to s-dimensional manifold

$$\mathbf{q} = \mathbf{q}(\sigma), \;\; \sigma \in \mathbb{R}^s$$

- Hand velocities belong to tangent bundle

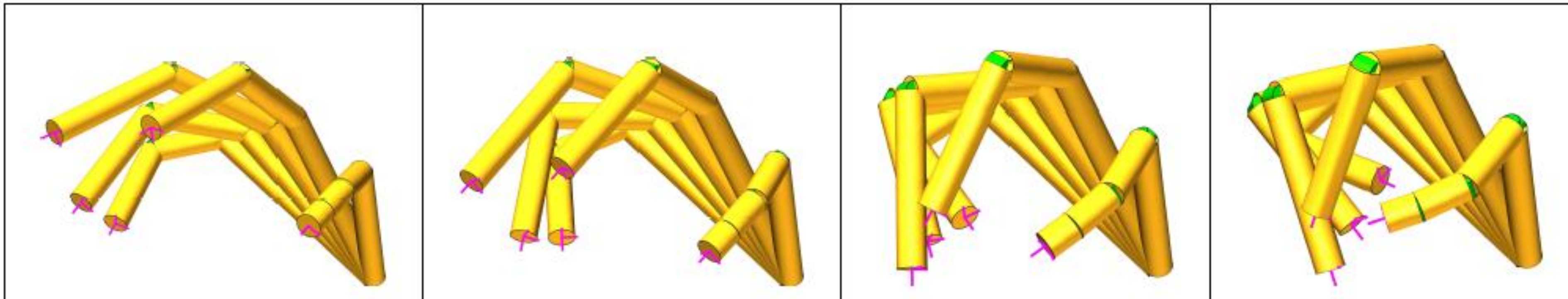
$$\dot{\mathbf{q}} = \mathbf{S}(\mathbf{q})\dot{\sigma}, \;\; \mathbf{S}(\cdot) \in \mathbb{R}^{n \times s}$$

- Fingers move according to Hand jacobian

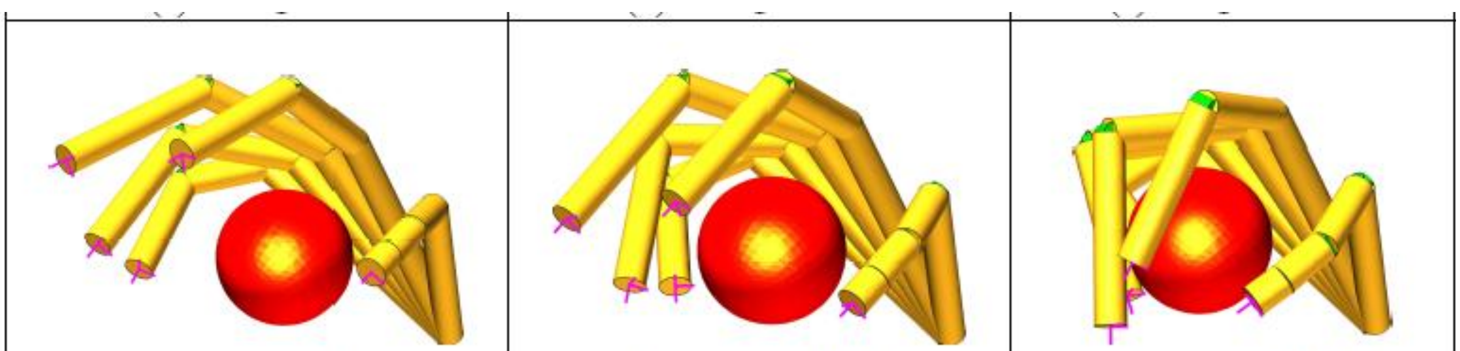
$$\dot{\mathbf{c}}_f = \mathbf{J}\dot{\mathbf{q}} = \mathbf{J}\mathbf{S}\dot{\sigma}$$

From Pre-Grasp to Grasp

- First synergy only



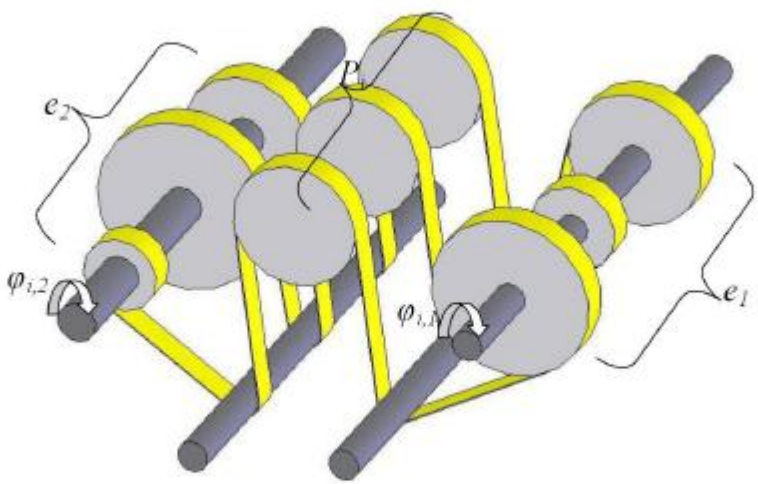
(a) $\sigma_1 = 0$ (b) $\sigma_1 = 0.35$ (c) $\sigma_1 = 0.70$ (d) $\sigma_1 = 1.0$

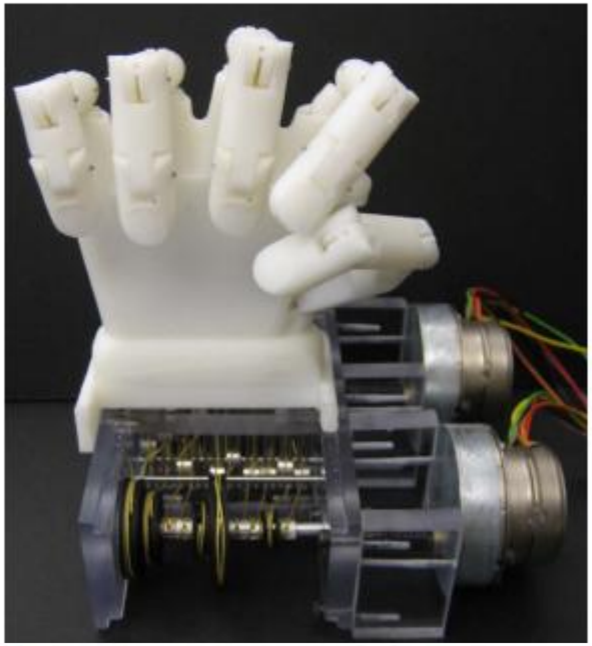


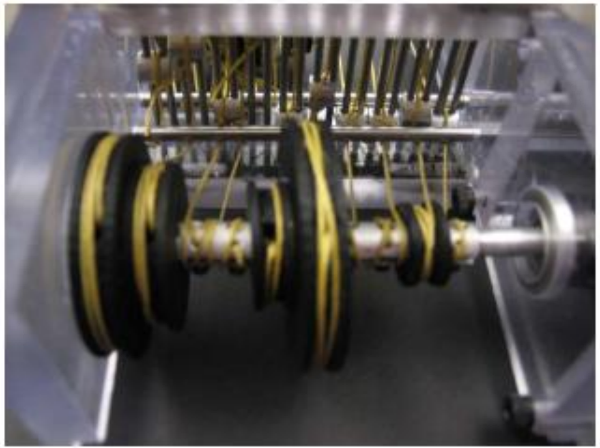
(e) $\sigma_1 = 0$ (f) $\sigma_1 = 0.35$ (g) $\sigma_1 = 0.70$

Fixes

- Stop on contact



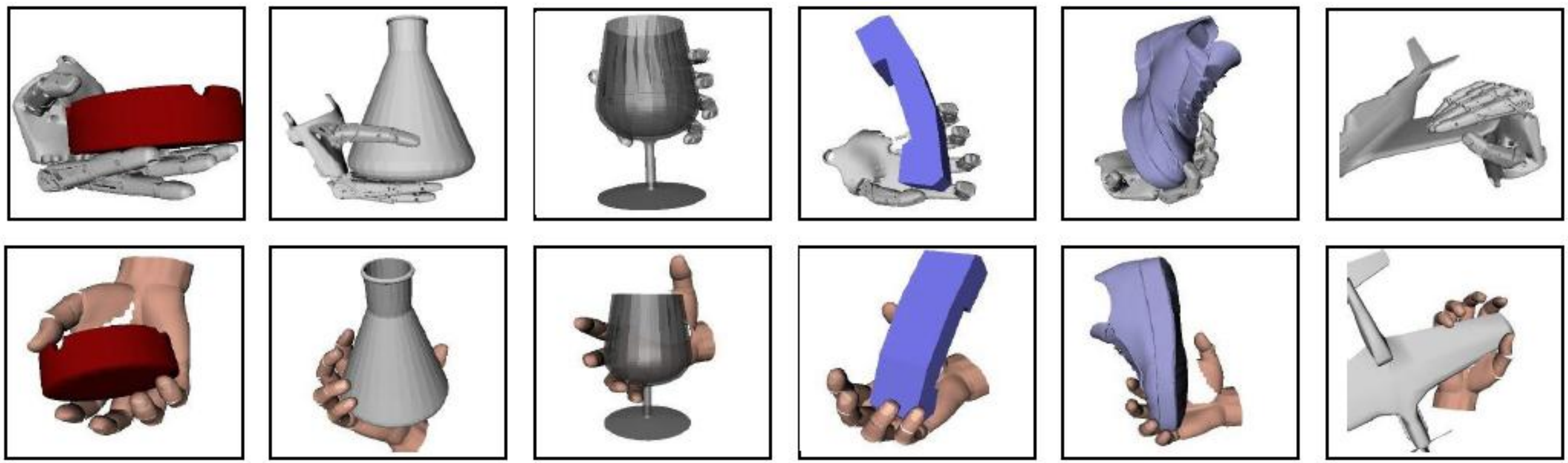




[Brown and Asada, IROS2007]

Fixes

- Stop and reshape the synergy (Ciocarlie *et al., 2007)*



- OK for grasp acquisition
- Not for force distribution analysis/control

Grasp Force Distribution

- Key point in grasping is *how forces are applied and controlled*
- Geometric synergy model can not account for grasping force generation
- What relation is there between grasping forces, grasping postures, and synergies?

Force Distribution: Notation and Equations

- External load (wrench) $\mathbf{w}$
- Grasp matrix $\mathbf{G}$ (*fat*)
- Contact forces $\mathbf{p}$

$$\mathbf{w} = \mathbf{G}\mathbf{p},$$

- Friction Constraints

$$\sigma_{i,f}(\mathbf{p}_i) = \alpha_i \|\mathbf{p}_i\| - \mathbf{p}_i^T \mathbf{n}_i < 0$$

- Given $\mathbf{w}$ which $\mathbf{p}$?

$$\mathbf{p} = \mathbf{G}^R \mathbf{w} + \mathbf{A}\mathbf{x},$$

- $\mathbf{G}^R$ (any) right inverse of $\mathbf{G}$
- $\mathbf{A}$: a basis of the subspace of internal (*squeezing*) forces
- By changing $\mathbf{x}$, squeezing forces are changed: if for every $\mathbf{w}$ it is possible to find $\mathbf{x}$ such that friction constraints are verified, than one has FcC
- **This only holds for fingertip grasping with a large number of synergies!**

Grasping with Synergies

- Hand joint torques τ
- Hand Jacobian $\mathbf{J}$
- Hand with synergies

$$\tau = \mathbf{J}^T \mathbf{p},$$

$$\tau_\sigma = \mathbf{S}^T \mathbf{J}^T \mathbf{p}$$

$$\mathbf{S}^T \mathbf{J}^T \in \mathbb{R}^{s \times p}$$

$$s < p$$

- **Not invertible in general → can not apply arbitrary contact forces $\mathbf{p}$!**

Grasping with Synergies

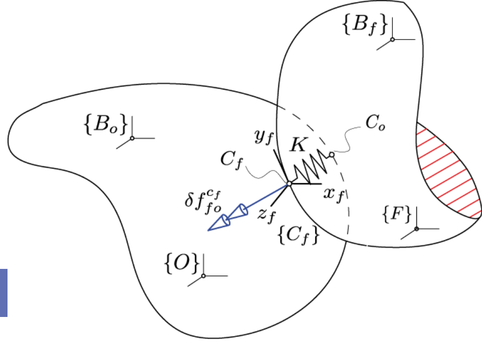
5 contact points

15 components of (frictional) contact forces

10 joints

1 synergy…

Grasping Forces & Synergies



- Q: What internal forces at equilibrium are modifiable at will in a given grasp?

$$\mathbf{w} = \mathbf{G}\mathbf{p},$$

$$\tau_\sigma = \mathbf{S}^T \mathbf{J}^T \mathbf{p}$$

The rigid-body model of grasp is statically indeterminate – no way to determine $\mathbf{p}$ for given $\mathbf{w}$ and τ!

- Must introduce congruence and constitutive equations – i.e. compliance

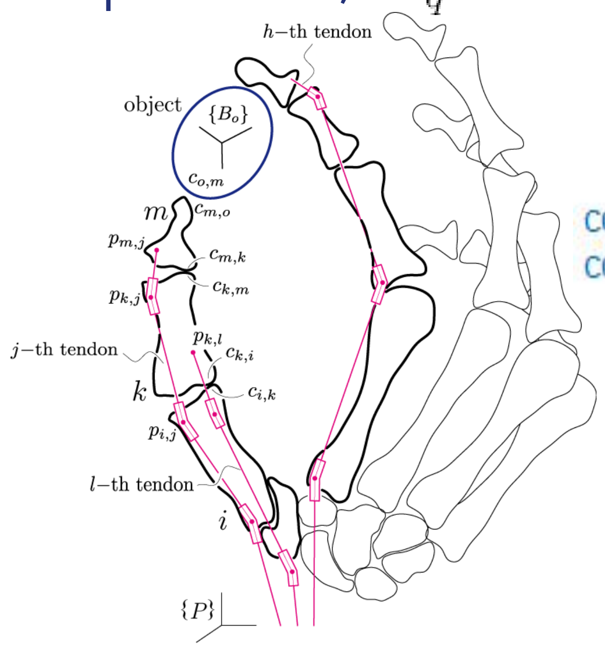
$$\dot{\mathbf{c}}_o = \mathbf{G}^T \dot{\mathbf{u}}$$

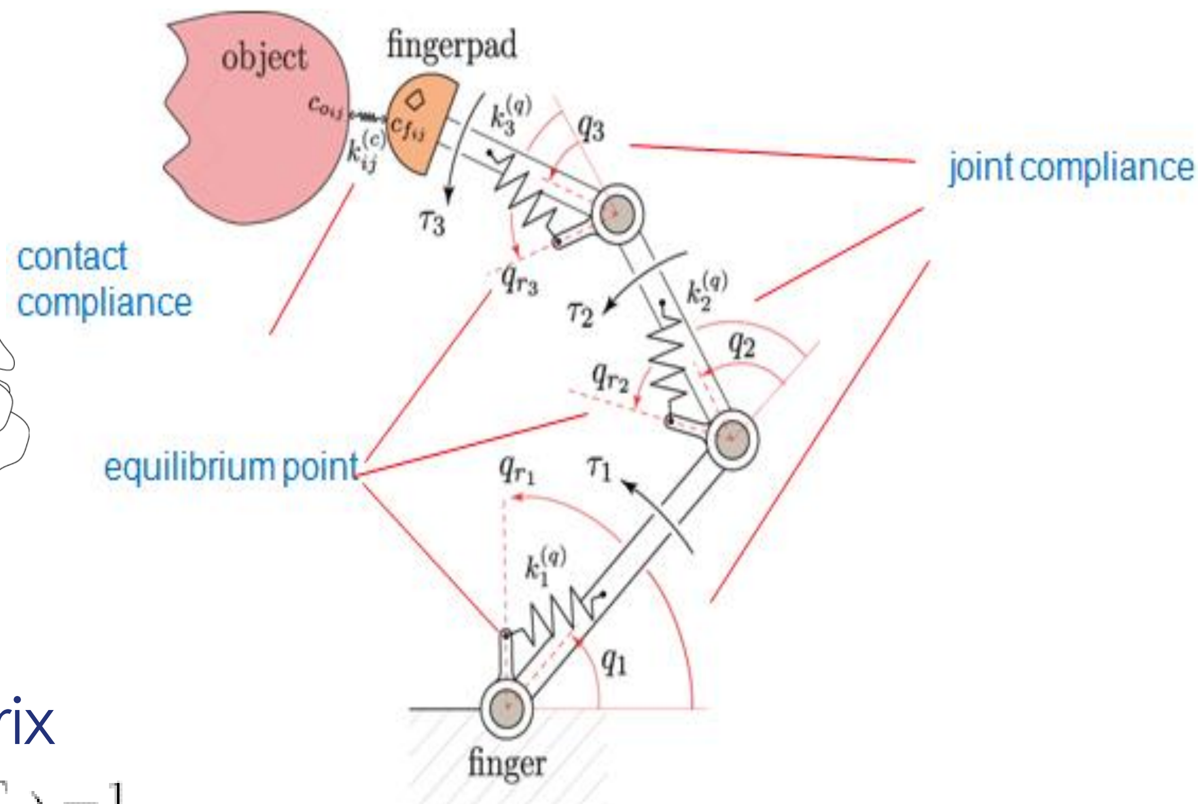
$$\dot{\mathbf{c}}_f = \mathbf{J}\mathbf{S}\ \dot{\sigma}$$

$$\mathbf{p} = \mathbf{K}\left(\mathbf{c}_f - \mathbf{c}_o\right)$$

Synergies and Hand Compliance

- The stiffness matrix of a hand takes into account both structural compliance at the contacts, $\mathbf{C}_s$, and joint-level compliance, $\mathbf{C}_q$





Overall stiffness matrix

$$\mathbf{K} = \left(\mathbf{C}_s + \mathbf{J}\mathbf{C}_q\mathbf{J}^{\mathrm{T}}\right)^{-1}$$

Internal Forces in Grasping with Synergies

- Internal Forces: $\mathbf{p} \in ker(\mathbf{G})$
- Not all internal forces are independently controllable acting through the joints

TH: The set of contact forces which can be actively controlled is a linear subspace of $ker(\mathbf{G})$

- $$\mathbf{A}\mathbf{x} = \mathbf{KJS}\Delta\sigma - \mathbf{KG}^T\Delta\mathbf{u}$$

  PLV →

  $$\begin{bmatrix}\mathbf{A} & -\mathbf{KJS} & \mathbf{KG}^T\end{bmatrix}\begin{pmatrix}\mathbf{x}\\ \Delta\sigma \\ \Delta\mathbf{u}\end{pmatrix} = 0.$$

- hence

  $$\mathbf{p}_a = (\mathbf{I} - \mathbf{G}_K^R\mathbf{G})\mathbf{KJS}\Delta\sigma$$

- or

  $$\boxed{\mathbf{p}_a = \mathbf{E}_\sigma\mathbf{y}}$$

The Soft Synergy Paradigm

- The posture (kinetic) synergy model only rules an internal “reference” representation of the hand configuration
- The higher level control of the hand commands this internal representation within the synergy manifold
- The actuator system of the hand is controlled towards this *reference hand* set-point
- The hand fingers and palm interact with manipulated objects and environment through contact
- The *physical hand* reaches an equilibrium under the effect of
  - attraction towards the synergy-driven reference hand
  - repulsion by contact forces
  - stiffness of actuators, tendons, and deformable bodies

Internal Forces in Grasping with Synergies

- Internal Forces: $\mathbf{p} \in ker(\mathbf{G})$
- Not all internal forces are active (controllable) acting on the joints

TH: The set of contact forces which can be actively controlled is a linear subspace of $ker(\mathbf{G})$

- $$\mathbf{A}\mathbf{x} = \mathbf{K}\mathbf{J}\mathbf{S}\Delta\sigma - \mathbf{K}\mathbf{G}^T\Delta\mathbf{u}$$

  PLV →

  $$\begin{bmatrix}\mathbf{A} & -\mathbf{K}\mathbf{J}\mathbf{S} & \mathbf{K}\mathbf{G}^T\end{bmatrix}\begin{pmatrix}\mathbf{x}\\ \Delta\sigma \\ \Delta\mathbf{u}\end{pmatrix} = 0.$$

- hence

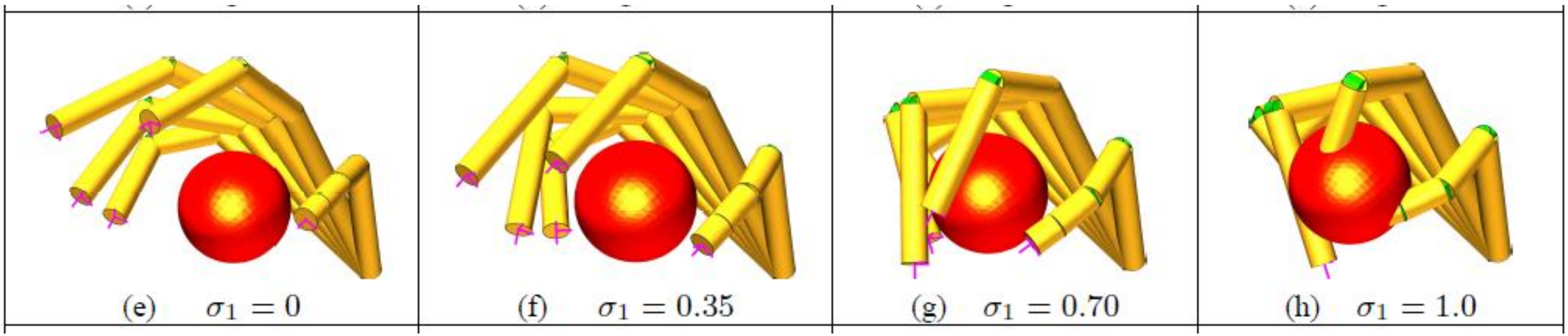
  $$\mathbf{p}_a = (\mathbf{I} - \mathbf{G}_K^R\mathbf{G})\mathbf{K}\mathbf{J}\mathbf{S}\Delta\sigma$$

- or $$\mathbf{p}_a = \mathbf{E}_\sigma \mathbf{y}$$

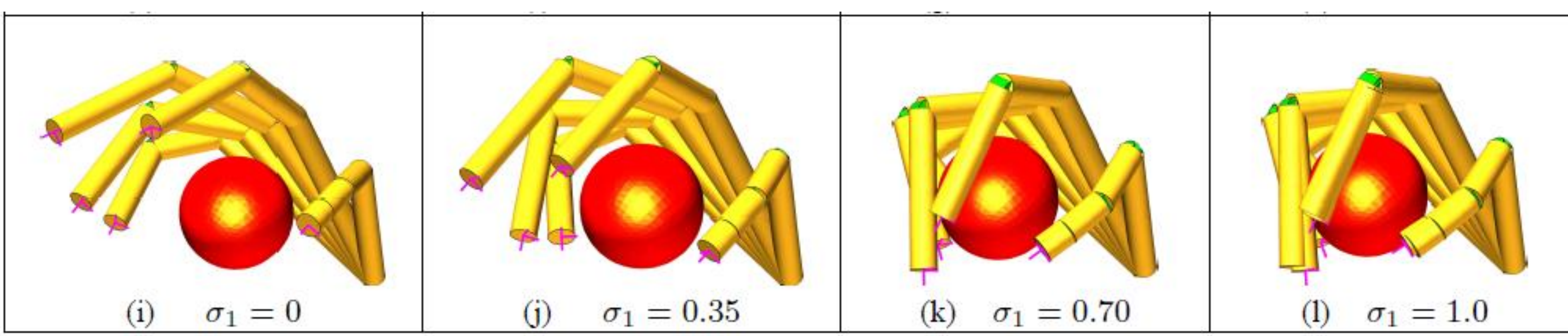
λ?

Visualizing Soft Synergies

- Rigid Synergy = Reference Hand

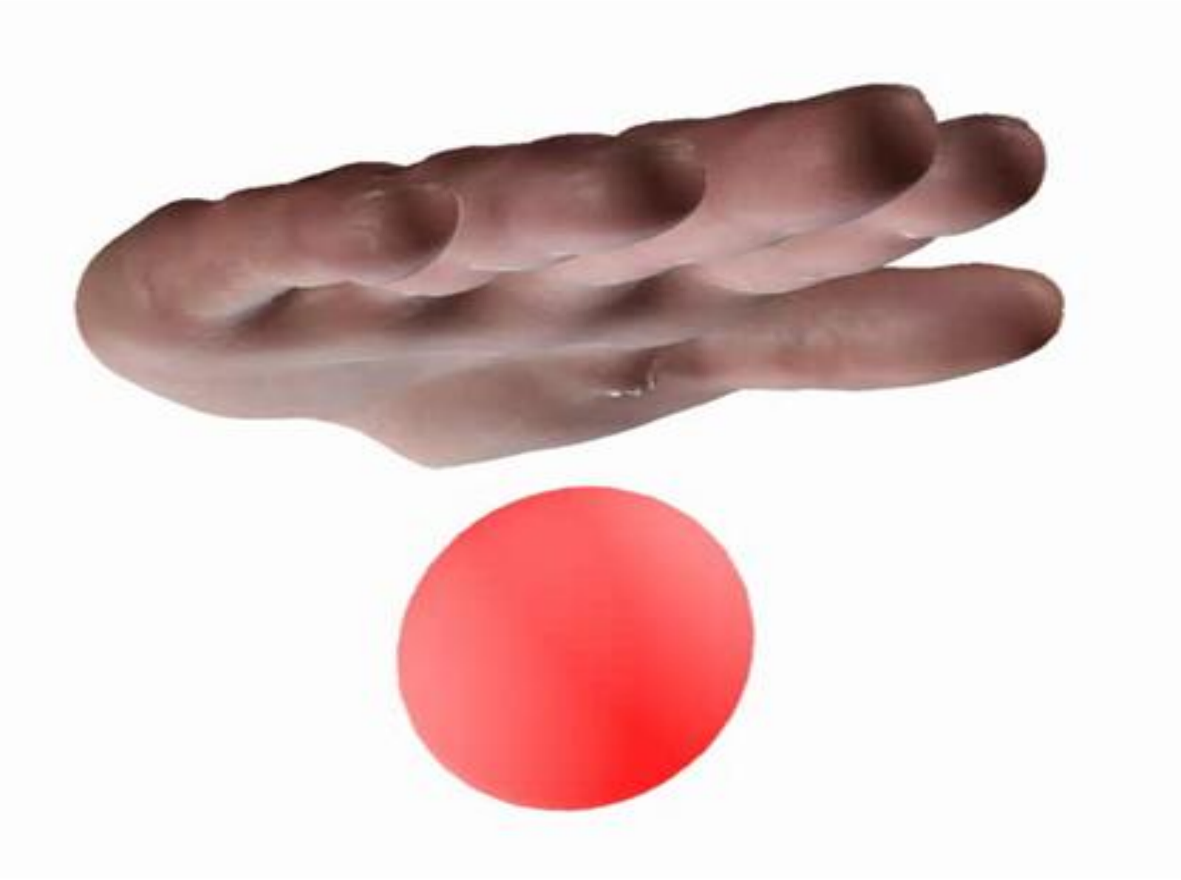


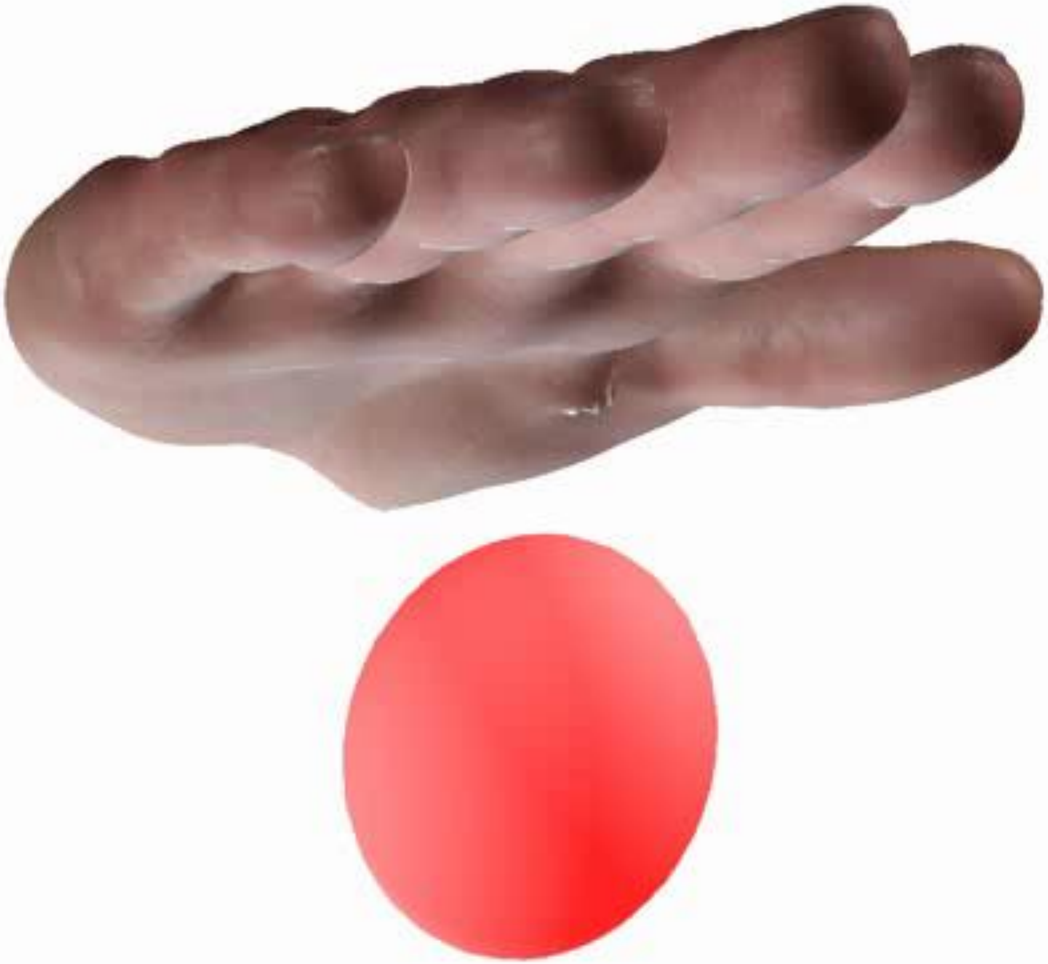
- Soft Synergy = Equilibrium Hand



Pinch Grasping with 3 soft Synergies

- Cherry





Power Grasping with 3 soft Synergies

- Ashtray

Questions

The first few synergies of the human hand explain much of covariance of grasp approach.

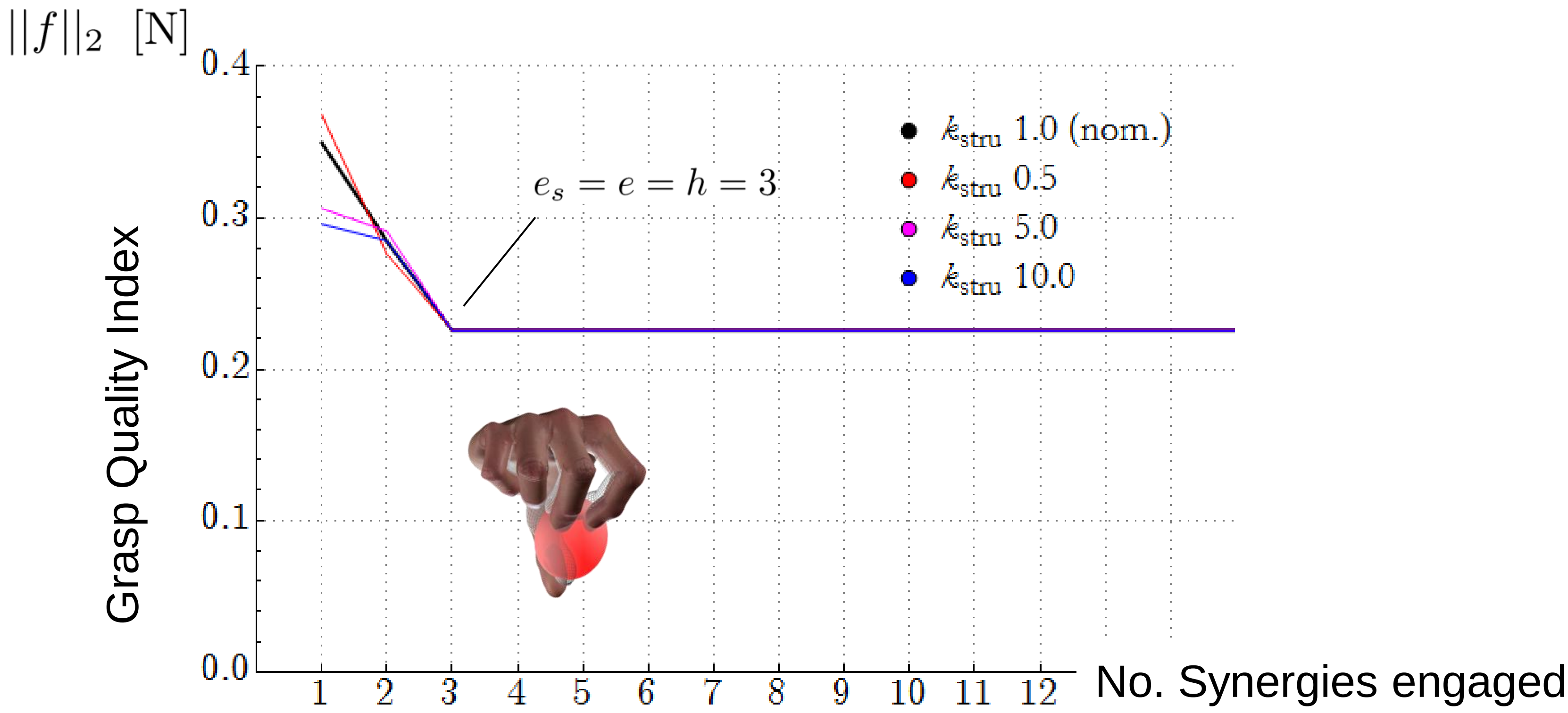
- Are these also dominating in grasping forces? (no experimental data so far)

Study of grasping forces depends on hand compliance.

- How robust are the results?

Precision grasp

- Variation of grasp quality measure with # synergies engaged in grasp
- Dimension of Internal Force subspace: 3
- Grasp is already force-closure using the 1-st synergy only
- Negligible effect of contact stiffness variation



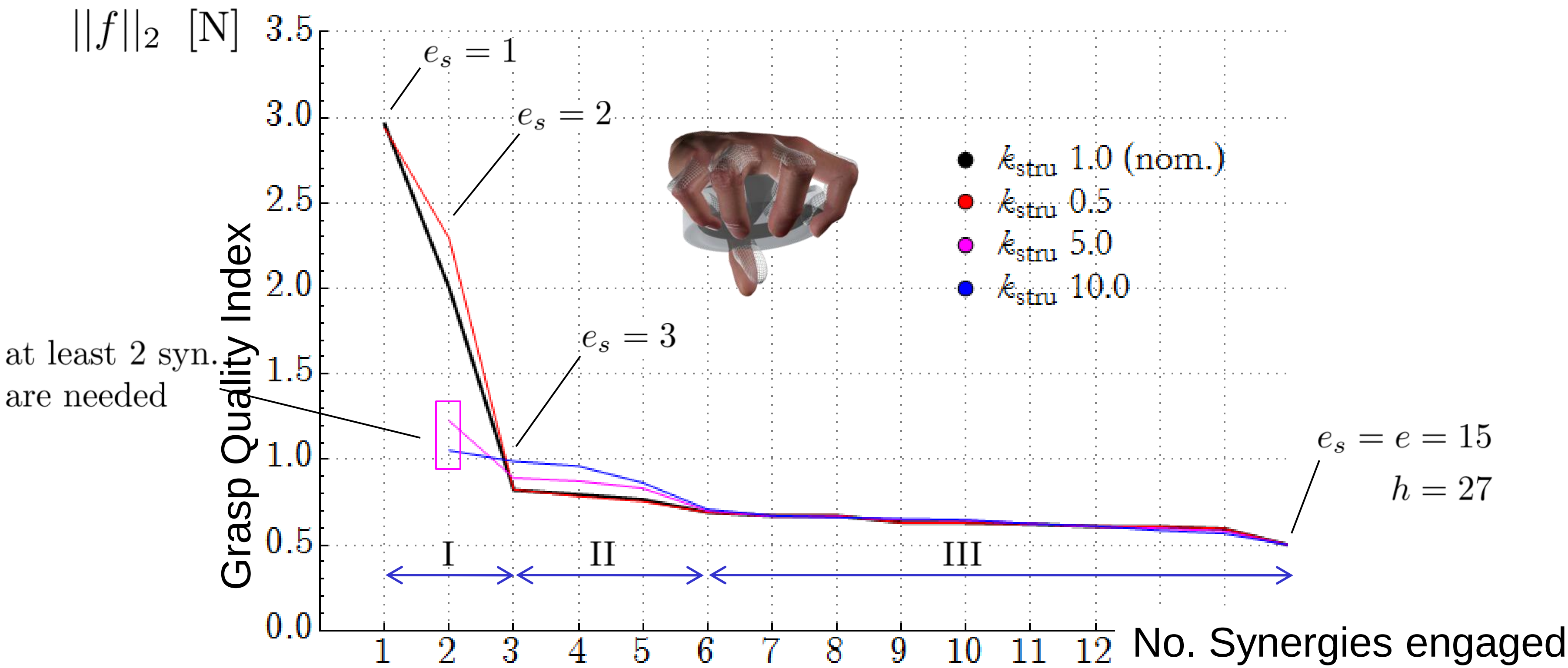
Control experiment

- Randomized the order of synergies:

Synergies Engaged	Force closure	Grasp Quality
7	NO	-
7,4	NO	-
7,4,12	NO	-
7,4,12,5	NO	-
7,4,12,5,8	NO	-
7,4,12,5,8,1	OK	1.37
7,4,12,5,8,1,4	OK	1.37
7,4,12,5,8,1,4,2	OK	1.34

Power grasp

- Variation of grasp quality measure with # synergies engaged in grasp
- Dimension of Internal Force subspace: 27
- Grasp is not always force-closure with the 1-st synergy only
- Limited effect of contact stiffness variation



Summary

- Analytical Results
  - Translating synergies form kinematic to force domain (soft-synergies).
  - Solve the force decomposition and optimization problem for hands with synergies;
- “Experimental” Results
  - force-closure properties of grasps strongly depends on which synergies are used to control the hand;
  - if the first few synergies (PCs) are not actively controlled, force-closure can be obtained only by many more DoFs;
  - quality of grasp (different norms of contact forces to prevent slippage) is enhanced by increasing the actuated synergies, but only to a limited extent;
  - no improvement beyond the first three synergies for precision grasp, continuous but small improvements in the whole-hand grasp case;
  - results are consistently robust with respect to different values of the stiffness parameters (uncertainty in their knowledge/control).

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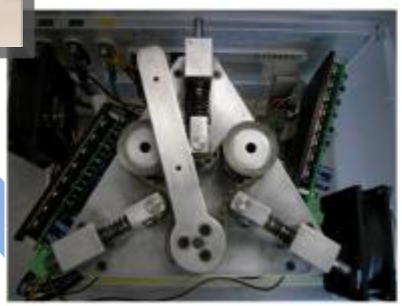
- A bit of a retrospective
- Using Variable Impedance
  - Optimal control
    - Safety oriented
    - Performance oriented
    - Energy oriented
  - Tele-Impedance
- Measuring Variable Impedance
- VSA and Hands
- Design

Variable Stiffness Actuators – Pisa

Soft Arm: 2000



VSA I: 2003

VSA II: 2008

VSA HD: 2010

QBots: TODAY

VSA CUBE

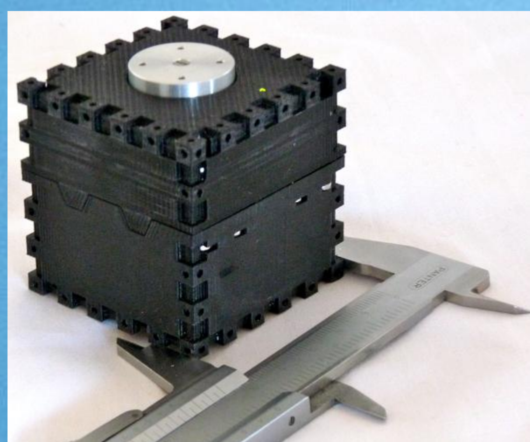
modular **servo** variable stiffness actuator

yesterday

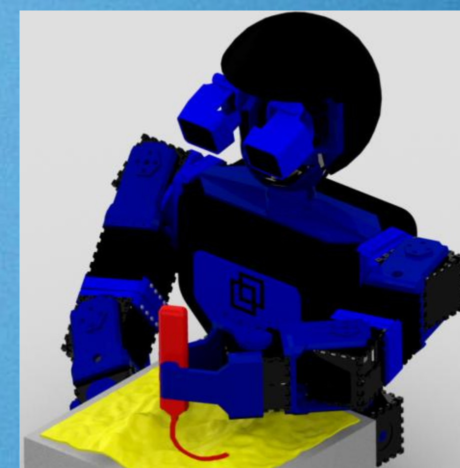
today

tomorrow

embedded
position control



embedded
position
and stiffness
control



Datasheet Template

Actuator Name

Typology (optional)

Fig.1 Picture

Fig.2 Mechanical interface drawings

Operating Data				
#	(quantity)		(unit)	(value)
Mechanical				
1	Continuous Output Power		[W]	xxx
2	Nominal Torque		[Nm]	xxx
3	Nominal Speed		[rad/s]	xxx
4	Nominal Stiffness Variation Time	with no load	[s]	xxx
5		with nominal torque	[s]	xxx
6	Peak (Maximum) Torque		[Nm]	xxx
7	Maximum Speed		[rad/s]	xxx
8	Maximum Stiffness		[Nm/rad]	xxx
9	Minimum Stiffness		[Nm/rad]	xxx
10	Maximum Elastic Energy		[J]	xxx
11	Maximum Hysteresis		[°]	xxx
12	Maximum deflection	with max. stiffness	[°]	xxx
13		with min. stiffness	[°]	xxx
14	Active Rotation Angle		[°]	xxx
15	Angular Resolution		[°]	xxx
16	Weight		[Kg]	xxx
Electrical				
17	Nominal Voltage		[V]	xxx
18	Nominal Current		[A]	xxx
19	Maximum Current		[A]	xxx
Control				
20	Voltage Supply		[V]	xxx
21	Nominal Current		[A]	xxx
22	I/O protocol		[]	xxx

Fig.3 Stiffness vs Torque

Fig.4 Speed vs Torque

Fig.5 Deflection vs Torque

Fig.6 Connection diagram

Actuator Name (repetition)

Additional Characteristics

Fig.7 Measured Torque vs Deflection

Fig.8 3D workspace

Additional sensors data			
#	(quantity)	(unit)	(value)
a0	**Sensor a**		
a1	Resolution	[yyy]	xxx
a2	Range	[yyy]	xxx
a3	I/O protocol	[yyy]	xxx
ax	(specific sensor properties)	[yyy]	xxx
b0	**Sensor b**		
bx	(specific sensor properties)	[yyy]	xxx
by	(specific sensor properties)	[yyy]	xxx
bz	(specific sensor properties)	[yyy]	xxx
n0	**Sensor n**		
...	...	...	...

Fig.9 Sensor Map

This space is left blank for any integrative information at the compiler's discretion. Examples may include:

- additional system images
- max. structural load values
- accessories
- software details

Actuator Name (repetition)

Model

Fig.10a Actuator Internals Layout

Fig.10b Actuator Internals Working Principle

Mathematical model		
101	Recoil Point Function	$x_e = x_e(q_1, q_2)$
102	Energy Function	$H = H(q_1, q_2, x)$
103	Output Torque Function	$\tau = \tau(q_1, q_2, x)$
104	Output Stiffness Function	$\sigma = \sigma(q_1, q_2, x)$
105	Spring Torque Function	$e_S = e_S(q_1, q_2, x)$
106	Springs to Motors Transmission Ratio	$A = A(q_1, q_2, x)$
107	Springs to Output Transmission Ratio	$B = B(q_1, q_2, x)$

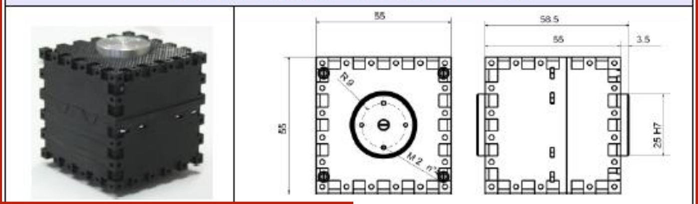
Standard VS template

A result of a combined effort by UNIPI, DLR and IIT

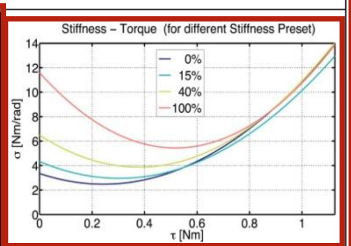
Developed inside the VIACTORS project

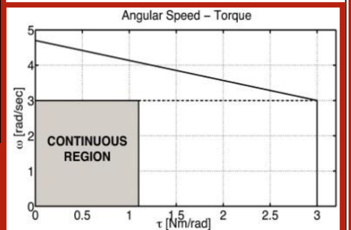
VSA CUBE

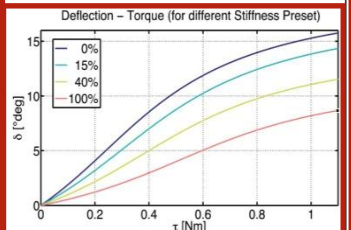
Bidirectional Agonistic - Antagonistic

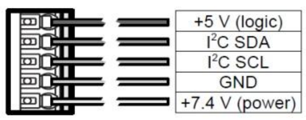


Operating Data				
#	(quantity)		(unit)	(value)
Mechanical				
1	Continuous Output Power		[W]	3.3
2	Nominal Torque		[Nm]	1.1
3	Nominal Speed		[rad/s]	3
4	Nominal Stiffness Variation Time	with no load	[s]	0.18
5		with nominal torque	[s]	0.32
6	Peak (Maximum) Torque		[Nm]	3
7	Maximum Speed		[rad/s]	4.7
8	Maximum Stiffness		[Nm/rad]	14
9	Minimum Stiffness		[Nm/rad]	3
10	Maximum Elastic Energy		[J]	0.047
11	Maximum Hysteresis		[°]	2.5
12	Maximum deflection	with max. stiffness	[°]	8.6
13		with min. stiffness	[°]	15.8
14	Active Rotation Angle		[°]	120
15	Angular Resolution		[°]	0.175
16	Weight		[Kg]	0.260
Electrical				
17	Nominal Voltage		[V]	7.4
18	Nominal Current		[A]	2
19	Maximum Current		[A]	6
Control				
20	Voltage Supply		[V]	5
21	Nominal Current		[A]	0.2
22	I/O protocol		[]	I^2C









Operating Data			
#	(quantity)	(unit)	(value)
Mechanical			
1	Continuous Output Power	[W]	3.3
14	Active Rotation Angle	[°]	120
15	Angular Resolution	[°]	0.175
16	Weight	[Kg]	0.260

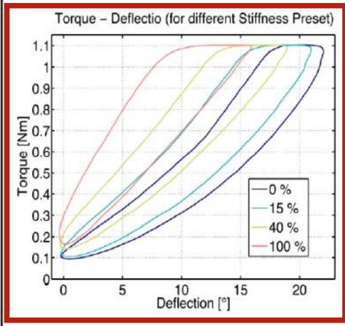
Stiffness – Torque (for different Stiffness Preset)

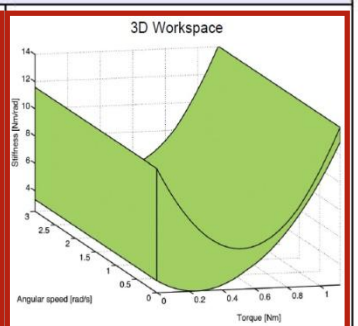
Electrical			
17	Nominal Voltage	[V]	7.4
18	Nominal Current	[A]	2
19	Maximum Current	[A]	6
Control			
20	Voltage Supply	[V]	5
21	Nominal Current	[A]	0.2
22	I/O protocol	[]	I^2C

τ [Nm]

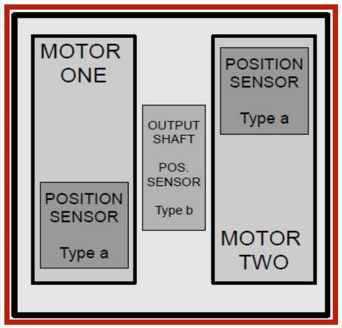
VSA CUBE

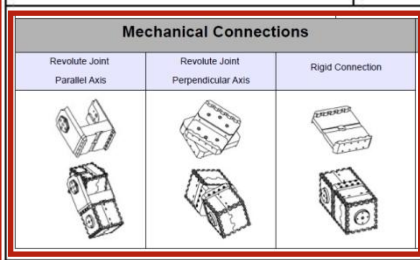
Bidirectional Agonistic - Antagonistic

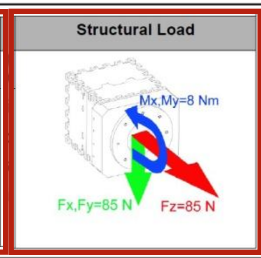


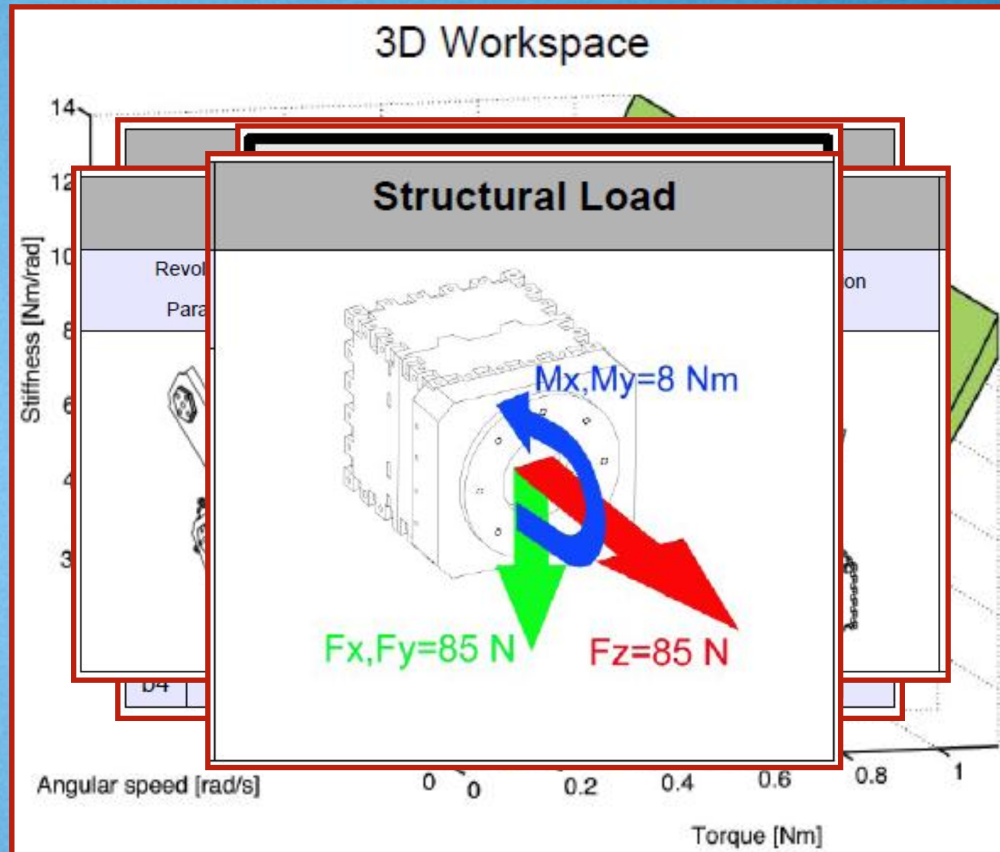


Additional sensors data			
#	(quantity)	(unit)	(value)
a0	Sensor a		
a1	Resolution	[°]	0,175
a2	Range	[°]	0 - 270
a3	I/O protocol	[]	Analog
ax	Voltage Supply	[V]	5
b0	Sensor b		
b1	Resolution	[°]	0,175
b2	Range	[°]	0 - 360
b3	I/O protocol	[]	Analog
b4	Voltage Supply	[V]	5



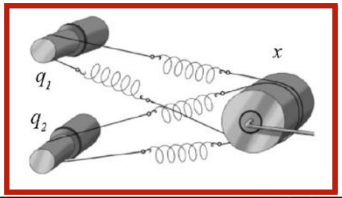


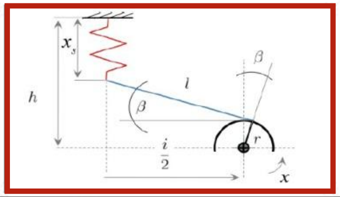




VSA CUBE

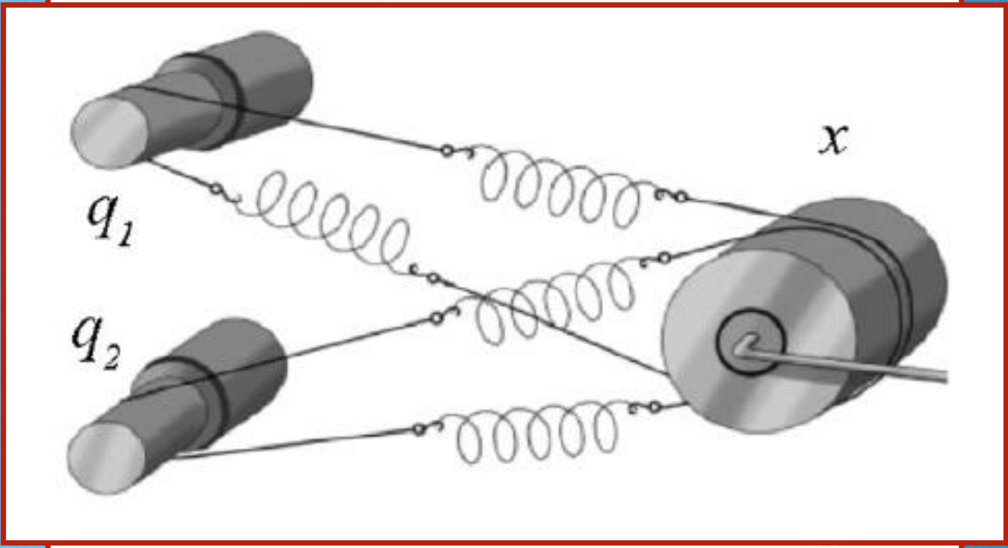
Bidirectional Agonistic - Antagonistic



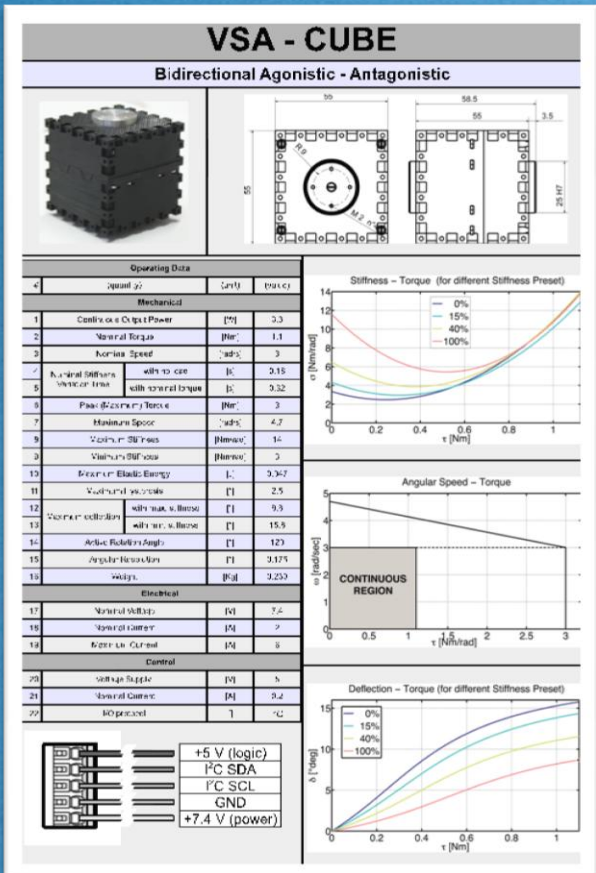


Mathematical model

101	Recoil Point Function	$x_e(q) = \frac{q_1 + q_2}{2}$
102	Energy Function	$H(q,x) = 0.00973 \left(\arcsin(2\lvert q_1 - x \rvert)^{2.1} + \arcsin(2\lvert q_2 - x\rvert)^{2.1} \right)$
103	Output Torque Function	$\tau(q,x) = 0.0407 \left(\frac{\arcsin(2(q_1 - x))^{1.09}}{\sqrt{1-4(q_1-x)^2}} + \frac{\arcsin(2(q_2 - x))^{1.09}}{\sqrt{1-4(q_2-x)^2}} \right)$
104	Output Stiffness Function	$\sigma(q,x) = 0.00973 \left(\left(\frac{9.11 \arcsin(2\lvert x - q_1\rvert)^{0.0896}}{1-4(x-q_1)^2} + \frac{16.7 \lvert x - q_1 \rvert \arcsin(2\lvert x-q_1\rvert)^{1.09}}{(1-4(x-q_1)^2)^{\frac{3}{2}}} \right) + \left(\frac{9.11 \arcsin(2\lvert x - q_2\rvert)^{0.0896}}{1-4(x-q_2)^2} + \frac{16.7 \lvert x - q_2 \rvert \arcsin(2\lvert x-q_2\rvert)^{1.09}}{(1-4(x-q_2)^2)^{\frac{3}{2}}} \right) \right)$
105	Spring Torque Function	$e_s(q,x) = \begin{bmatrix} -\frac{0.0407 \arcsin(2(q_1 - x))^{1.09}}{\sqrt{1-4(q_1-x)^2}} \\ -\frac{0.0407 \arcsin(2(q_2 - x))^{1.09}}{\sqrt{1-4q_2-x)^2}} \end{bmatrix}$
106	Springs to Motors Transmission Ratio	$A(q,x) = \begin{bmatrix} \frac{0.00652 \arcsin(2(q_1-x))^{0.0448}}{\sqrt{1-4(q_1-x)^2}} & 0 \\ 0 & \frac{0.00652 \arcsin(2(q_2-x))^{0.0448}}{\sqrt{1-4(q_2-x)^2}} \end{bmatrix}$
107	Springs to Output Transmission Ratio	$B(q,x) = \begin{bmatrix} -\frac{0.00652 \arcsin(2(q_1-x))^{0.0448}}{\sqrt{1-4(q_1-x)^2}} \\ -\frac{0.00652 \arcsin(2(q_2-x))^{0.0448}}{\sqrt{1-4(q_2-x)^2}} \end{bmatrix}$



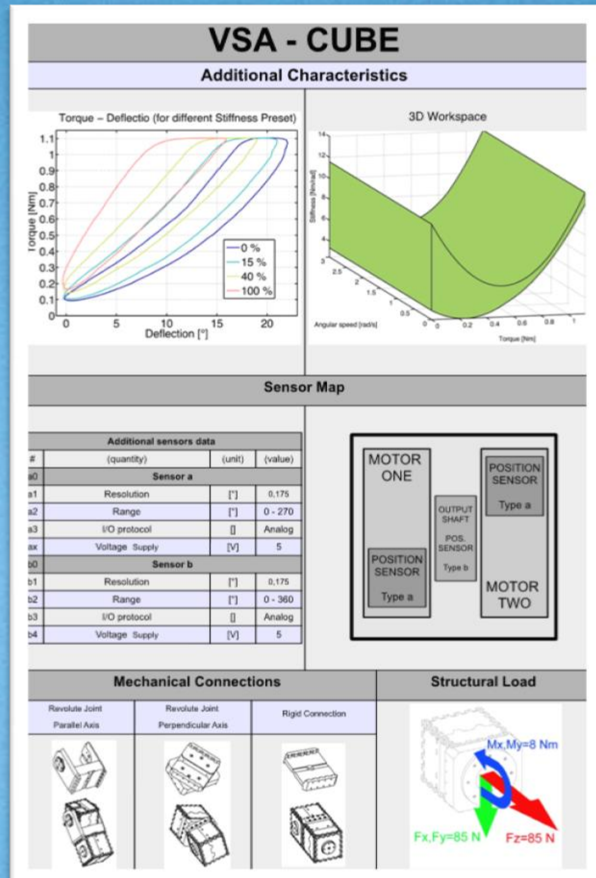
Cubebot datasheet



VSA - CUBE

Bidirectional Agonistic - Antagonistic

#	(quantity)	(unit)	(value)
	Mechanical		
1	Continuous Output Power	[W]	3.3
2	Nominal Torque	[Nm]	1.1
3	Nominal Speed	[rad/s]	3
4	Nominal Stiffness, without torque	[-]	0.15
5	Nominal Stiffness, with nominal torque	[-]	0.32
6	Peak (Maximum) Torque	[Nm]	3
7	Maximum Speed	[rad/s]	4.7
8	Maximum Stiffness	[Nm/rad]	14
9	Minimum Stiffness	[Nm/rad]	3
10	Maximum Elastic Energy	[J]	0.047
11	Maximum Deflection	[°]	2.5
12	Maximum deflection, with max. stiffness	[°]	9.3
13	Maximum deflection, with min. stiffness	[°]	15.9
14	Active Rotation Angle	[°]	120
15	Angular Resolution	[°]	0.175
16	Weight	[Kg]	0.250
	Electrical		
17	Nominal Voltage	[V]	7.4
18	Nominal Current	[A]	2
19	Maximum Current	[A]	5
	Control		
20	Voltage Supply	[V]	5
21	Nominal Current	[A]	0.2
22	I/O protocol	[]	I²C



VSA - CUBE

Additional Characteristics

Sensor Map

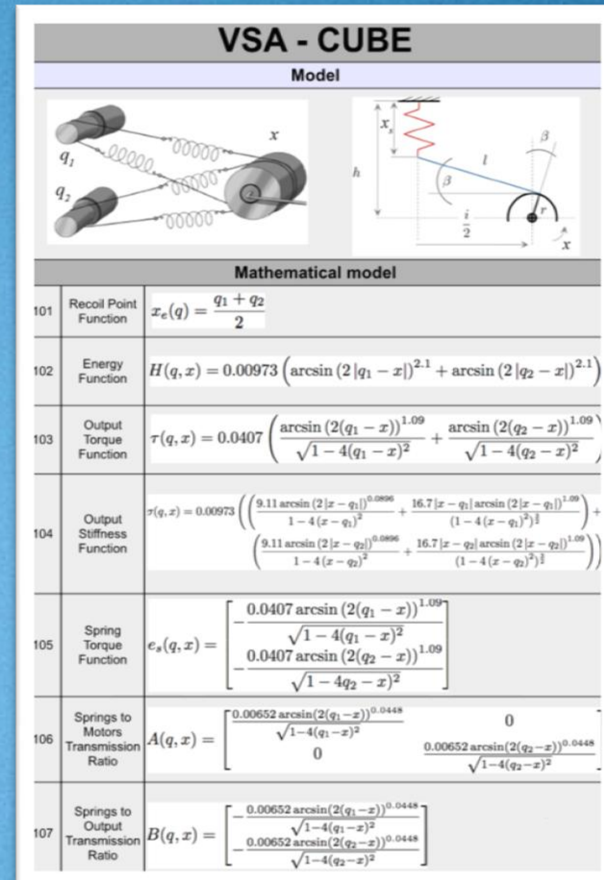
Additional sensors data

#	(quantity)	(unit)	(value)
a0	**Sensor a**		
a1	Resolution	[°]	0.175
a2	Range	[°]	0 - 270
a3	I/O protocol	[]	Analog
ax	Voltage Supply	[V]	5
b0	**Sensor b**		
b1	Resolution	[°]	0.175
b2	Range	[°]	0 - 360
b3	I/O protocol	[]	Analog
b4	Voltage Supply	[V]	5

Mechanical Connections

Revolute Joint Parallel Axis | Revolute Joint Perpendicular Axis | Rigid Connection

Structural Load



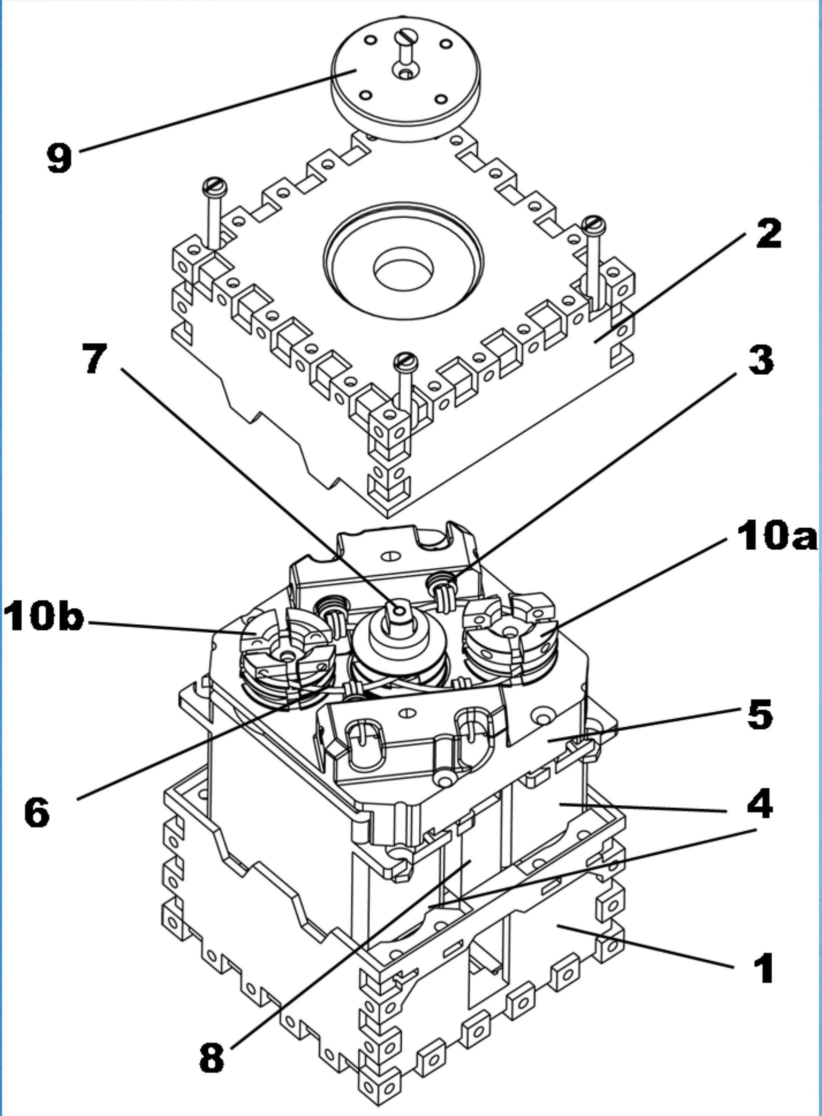
VSA - CUBE

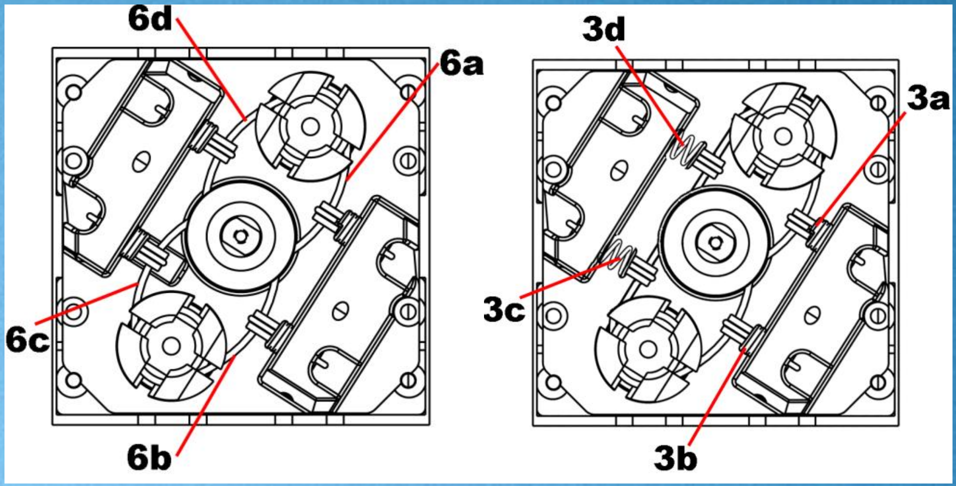
Model

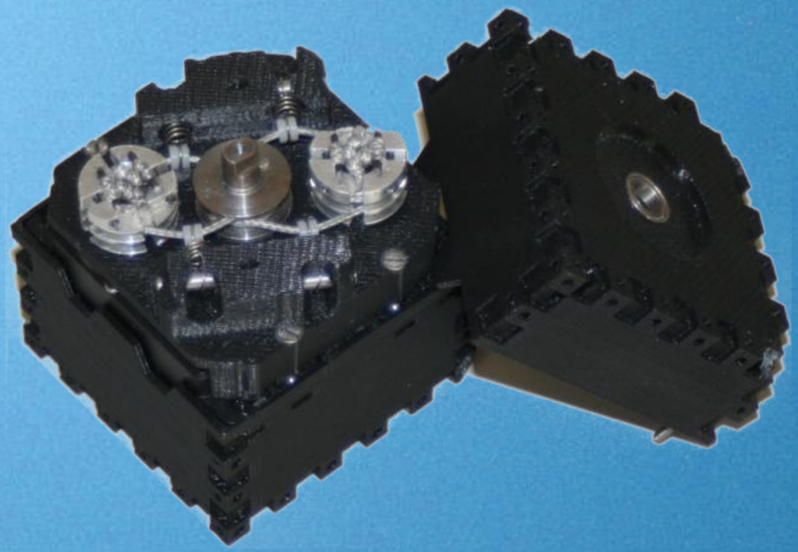
Mathematical model

#		
101	Recoil Point Function	$x_e(q) = \frac{q_1 + q_2}{2}$
102	Energy Function	$H(q,x) = 0.00973\left(\arcsin(2\lvert q_1 - x\rvert)^{2.1} + \arcsin(2\lvert q_2 - x\rvert)^{2.1}\right)$
103	Output Torque Function	$\tau(q,x) = 0.0407\left(\frac{\arcsin(2(q_1-x))^{1.09}}{\sqrt{1-4(q_1-x)^2}} + \frac{\arcsin(2(q_2-x))^{1.09}}{\sqrt{1-4(q_2-x)^2}}\right)$
104	Output Stiffness Function	$\sigma(q,x) = 0.00973\left(\left(\frac{9.11\arcsin(2\lvert x-q_1\rvert)^{0.0896}}{1-4(x-q_1)^2} + \frac{16.7\lvert x-q_1\rvert\arcsin(2\lvert x-q_1\rvert)^{1.09}}{(1-4(x-q_1)^2)^{\frac{3}{2}}}\right) + \left(\frac{9.11\arcsin(2\lvert x-q_2\rvert)^{0.0896}}{1-4(x-q_2)^2} + \frac{16.7\lvert x-q_2\rvert\arcsin(2\lvert x-q_2\rvert)^{1.09}}{(1-4(x-q_2)^2)^{\frac{3}{2}}}\right)\right)$
105	Spring Torque Function	$e_s(q,x) = \begin{bmatrix} -\frac{0.0407\arcsin(2(q_1-x))^{1.09}}{\sqrt{1-4(q_1-x)^2}} \\ -\frac{0.0407\arcsin(2(q_2-x))^{1.09}}{\sqrt{1-4(q_2-x)^2}} \end{bmatrix}$
106	Springs to Motors Transmission Ratio	$A(q,x) = \begin{bmatrix} \frac{0.00652\arcsin(2(q_1-x))^{0.0448}}{\sqrt{1-4(q_1-x)^2}} & 0 \\ 0 & \frac{0.00652\arcsin(2(q_2-x))^{0.0448}}{\sqrt{1-4(q_2-x)^2}} \end{bmatrix}$
107	Springs to Output Transmission Ratio	$B(q,x) = \begin{bmatrix} -\frac{0.00652\arcsin(2(q_1-x))^{0.0448}}{\sqrt{1-4(q_1-x)^2}} \\ -\frac{0.00652\arcsin(2(q_2-x))^{0.0448}}{\sqrt{1-4(q_2-x)^2}} \end{bmatrix}$

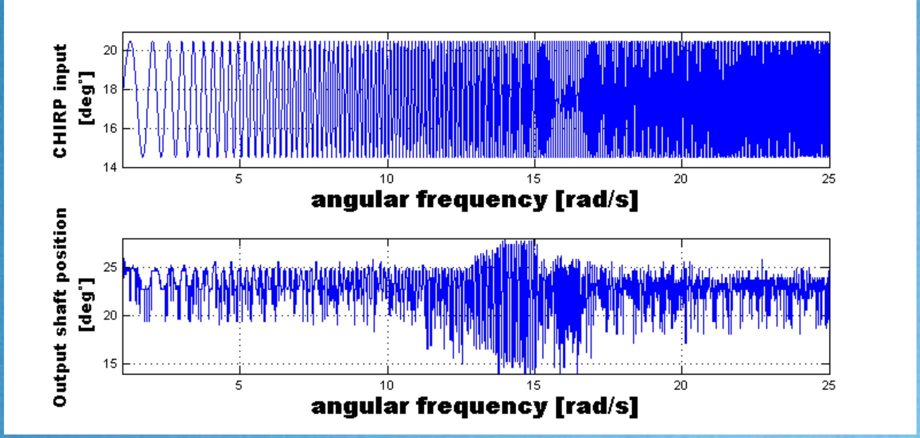
mechanics



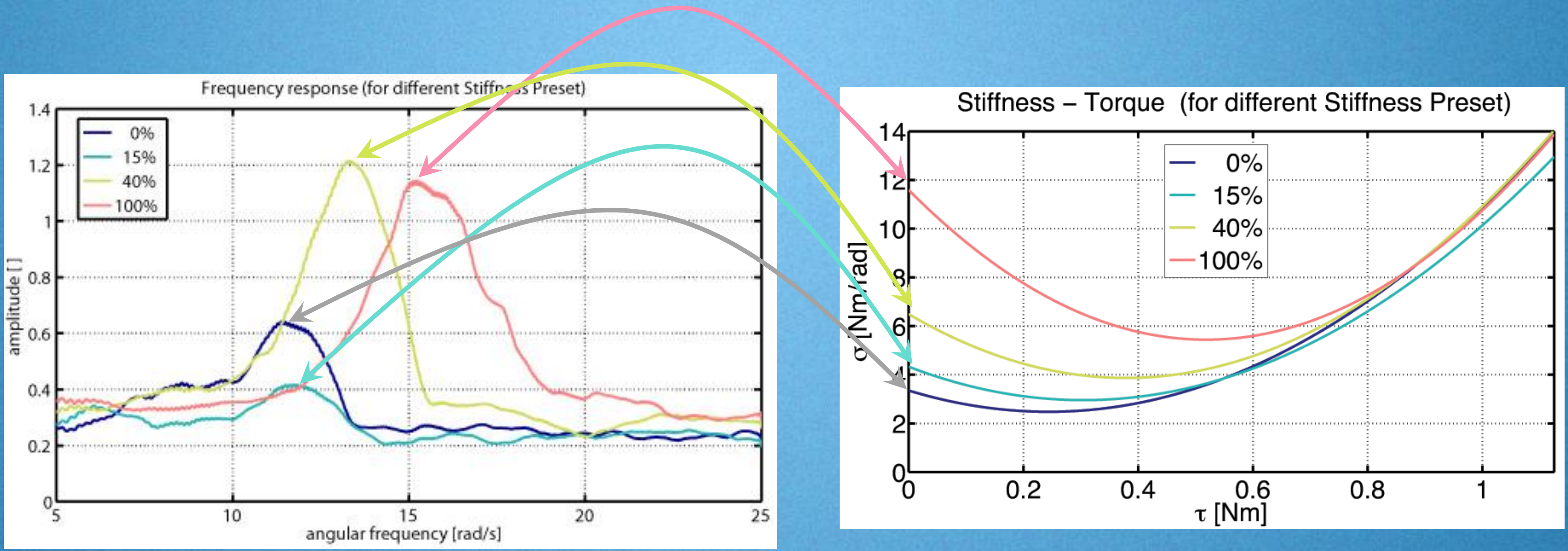


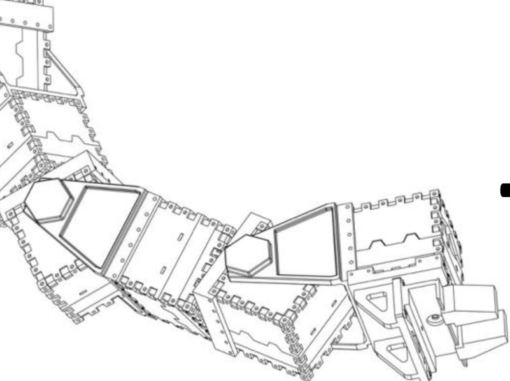


experimental tests



$$\omega = \sqrt{\frac{\sigma}{I}}$$

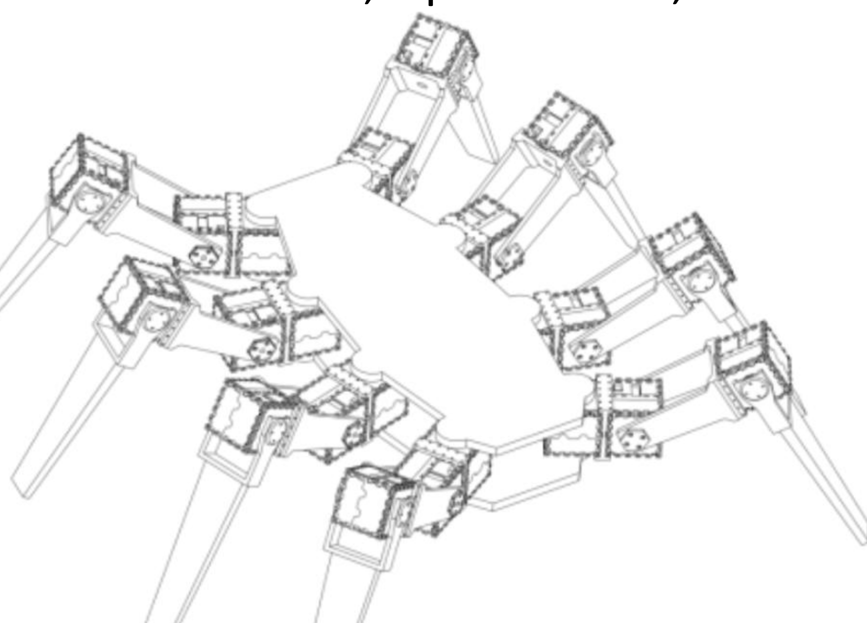


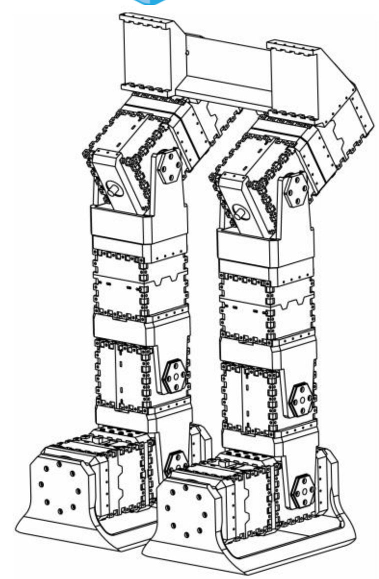


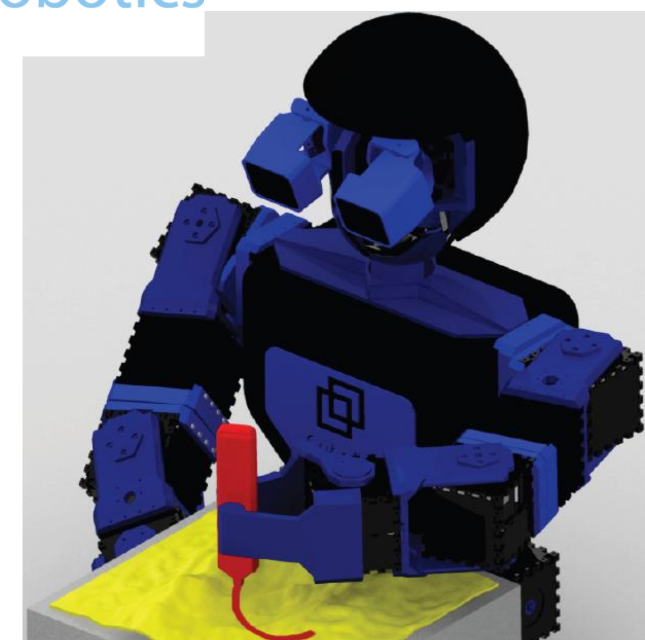
The Qbot

- ■ **Variable Stiffness Servo Motor**
  - ☐ Variable output shaft stiffness
  - ☐ Embedded position and stiffness regulation
  - ☐ Bus communication interface
  - ☐ Modular actuation and building unit
  - ☐ Low cost, Open-Source, Available

The Qboid







THANKS!