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Interaction Control for Contact Robotics

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Disclosure: Neville Hogan is part owner of Interactive Motion Technologies, Inc. which manufactures human-interactive technologies under license to MIT.

Outline

- Control Systems as Physical Systems
  - Information and Energy
  - Port Behavior (Impedance and Admittance)
  - An Equivalent Network for Interaction Control
- Some Implementations
  - Simple Impedance Control
  - Intrinsically Variable Inertia
  - Antagonist Actuators
- Stabilizing Interaction
  - Hamiltonian Analysis and Passivity
  - Force Feedback
- Some Applications
  - Biomimetic Artificial Arms
- Physical System Theory
  - Network Physical System Models
  - Nonlinear Equivalent Networks?

Coping with Contact

- The robots are coming!
  - Soon they'll be everywhere, rubbing shoulders with people
  - How do you design their controllers to **cope with contact**?
- A "signals" perspective permeates controller design
  - Pro: Operational modularity enables complex system design
  - Con: Limited to ***one-way*** interaction
- The core challenge of contact robotics:
  - Physical reality: interactions are ***two-way***
    - Contacted object dynamics couple to machine dynamics
    - Composite behavior is ***not*** a simple composition of operators
- An alternative approach:
  - Is it useful to ***describe control systems as physical systems***?

Information and Energy

- How can that help? Isn't a control system always a physical system?
- Physical dynamics process ***energy***, computers process ***information***
  - Computers and brains consume available energy, generate entropy, and get hot. That may limit their performance and speed, but…
  - Energy is largely ***irrelevant*** to what computers do
- Physical constraints on computation & signal processing
  - Temporal causality (no output before input)
  - Bounded variables (no infinite quantities)
  - …and that seems to be all
- Physical systems are (*much*) more constrained…
  - By the laws of mechanical physics—especially thermodynamics
    - A mechanical engineer's working definition

…and that may be used to advantage

Interaction Control

- Manipulation requires interaction
  - object behavior affects control of force and motion
- Independent control of force ***and*** motion is not possible
  - object behavior relates force and motion
    - contact a rigid surface: *kinematic* constraint
    - move an object: *dynamic* constraint
- Accurate control of force ***or*** motion requires detailed models of
    - manipulator dynamics
    - object dynamics
  - object dynamics are usually known poorly, often not at all
  - … one important example: a collaborating human

Object Behavior

- Can object forces be treated as external (exogenous) disturbances?
  - the usual assumptions don't apply:
    - "disturbance" forces aren't independent
    - forces often aren't small by any reasonable measure
- Can forces due to object behavior be treated as modeling uncertainties?
  - yes (to some extent) but the usual assumptions don't apply:
    - command and disturbance frequencies overlap
- Example: two people shaking hands
  - how each person moves influences the forces evoked
    - "disturbance" forces are state-dependent
  - each may exert comparable forces and move at comparable speeds
    - command & "disturbance" have comparable magnitude & frequency

Alternative: Control ***Port Behavior***

- Port behavior:
  - system properties and/or behaviors "seen" at an interaction port
- Interaction port:
  - characterized by conjugate variables that define power flow
- Key point:

$$\begin{cases} \text{power in} & P = \mathbf{e}^t \mathbf{f} \\ \mathbf{e} = [e_1 \cdots e_n]^t & \text{efforts (forces)} \\ \mathbf{f} = [f_1 \cdots f_n]^t & \text{flows (velocities)} \end{cases}$$

> port behavior is unaffected by contact and interaction

Impedance & Admittance

- Impedance and admittance characterize interaction
  - dynamic generalizations of resistance and conductance
  - introduced by Oliver Heaviside
- Usually introduced for linear systems but generalize to nonlinear systems
  - state-determined representation:
  - this form may be derived from or depicted as a ***network model***

electrical capacitor $Z(s) = \frac{e(s)}{i(s)} = \frac{1}{Cs}$

electrical inductor $Z(s) = \frac{e(s)}{i(s)} = L(s)$

$$\begin{cases} \dot{z} = Z_s(z,V) & \text{State equations} \\ F = Z_o(z,V) & \text{Output equations} \\ P = F^t V & \text{Constraint on input \& output} \\ z \in \Re^n, F \in \Re^m, V \in \Re^m, P \in \Re \end{cases}$$

Impedance & Admittance (continued)

- Admittance is the ***causal dual*** of impedance
  - Admittance: flow out, effort in
  - Impedance: effort out, flow in
- Linear system: admittance is the inverse of impedance
- Nonlinear system:
  - causal dual is well-defined:
  - but may not correspond to any impedance
    - the inverse may not exist

$Y(s) = Z(s)^{-1}$

electrical capacitor

$Y(s) = \frac{i(s)}{e(s)} = Cs$

$$\begin{cases} \dot{y} = Y_s(y,F) \\ V = Y_o(y,F) \\ P = F^t V \\ y \in \Re^n, F \in \Re^m, V \in \Re^m, P \in \Re \end{cases}$$

Impedance or Dynamic Stiffness?

- Impedance and admittance are ***port operators***
- Impedance may also be defined as a dynamic generalization of stiffness
  - effort out, displacement in
  - best for mechanical systems because of the key role of configuration (generalized position)

$$\begin{cases} \dot{z} = Z_s(z,X) \\ F = Z_o(z,X) \\ dW = F^t dX \\ z \in \Re^n, F \in \Re^m, X \in \Re^m, P \in \Re \end{cases}$$

- I prefer the general term "impedance" for ***any*** operator with motion in, effort out
- I prefer the general term "admittance" for ***any*** operator with effort in, motion out

Interaction Control: Causal Considerations

- What's the best input/output form for the manipulator?
- The set of objects likely to be manipulated includes
  - inertias
    - minimal model of most movable objects
  - kinematic constraints
    - simplest description of surface contact
- Causal considerations:
  - inertias *prefer* admittance causality
  - constraints *require* admittance causality
  - compatible manipulator behavior should be an impedance
- An **ideal** controller should try to make the manipulator behave as an impedance
  - Hence impedance control
    - Hogan 1979, 1980, 1985, etc.

The Challenge of a Child's Toy

E.D.Fasse & J.F.Broenink, U. Twente, NL

Network Modeling Perspective on Interaction Control

- Port concept
  - control interaction port behavior
  - **port behavior is unaffected by contact and interaction**
- Causal analysis
  - impedance and admittance characterize interaction
  - object is likely an admittance
  - (try to) control manipulator impedance
- Power exchange is possible with interaction
  - power sources are commonly modeled as equivalent networks
    - Thévenin equivalent
    - Norton equivalent
- Can equivalent network structure be applied to interaction control?

Nonlinear Equivalent Networks

- Can equivalent networks be defined for nonlinear systems?
  - Nonlinear impedance and admittance can be defined as above
  - Thévenin & Norton sources can also be defined
    - Hogan, N. (1985) *Impedance Control: An Approach to Manipulation.* ASME J. Dynamic Systems Measurement & Control, Vol. 107, pp. 1-24.
- However…
  - the simple connection is not guaranteed
- In other words:
  - separating the pieces is always possible
  - re-assembling them by superposition is not

Nonlinear Equivalent Network for Interaction Control

- One way to proceed:
  - specify an equivalent network structure in the (desired) interaction behavior
  - **provides key superposition properties**
- Specifically:
  - *nodic* desired impedance
    - does not require inertial reference frame
  - "virtual" trajectory
    - "virtual" as it need not be a realizable trajectory

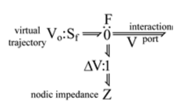
$\mathbf{V}_0 = \mathbf{V}_0 : \{c\}$ virtual trajectory

$\Delta\mathbf{V} = \mathbf{V}_0 - \mathbf{V}$

network junction structure (0 junction)

$\dot{\mathbf{z}} = \mathbf{Z}_s(\mathbf{z}, \Delta\mathbf{V}) : \{c\}$, $\mathbf{F} = \mathbf{Z}_o(\mathbf{z}, \Delta\mathbf{V}) : \{c\}$ } nodic impedance

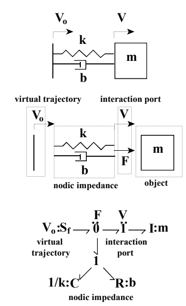
$:\{c\}$ denotes modulation by control inputs



Norton equivalent network

Virtual Trajectory

- Nodic impedance:
  - Defines desired interaction dynamics
  - Nodic because input velocity is defined relative to a "virtual" trajectory
- Virtual trajectory:
  - like a motion controller's reference or nominal trajectory ***but*** no assumption that dynamics are fast compared to motion
  - "virtual" because it need not be realizable
    - e.g., need not be confined to manipulator's workspace



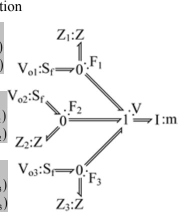
Superposition of "Impedance Forces"

- Minimal object model is an inertia
  - it responds to the sum of input forces
  - in network terms: it comes with an associated 1-junction
- This guarantees ***linear*** summation of component impedances…
- …even if the component impedances are ***nonlinear***

$\Delta\mathbf{V}_1 = \mathbf{V}_{o1} - \mathbf{V}$
$\dot{\mathbf{z}}_1 = \mathbf{Z}_{s1}(\mathbf{z}_1, \Delta\mathbf{V}_1)$
$\mathbf{F}_1 = \mathbf{Z}_{o1}(\mathbf{z}_1, \Delta\mathbf{V}_1)$

$\Delta\mathbf{V}_2 = \mathbf{V}_{o2} - \mathbf{V}$
$\dot{\mathbf{z}}_2 = \mathbf{Z}_{s2}(\mathbf{z}_2, \Delta\mathbf{V}_2)$
$\mathbf{F}_2 = \mathbf{Z}_{o2}(\mathbf{z}_2, \Delta\mathbf{V}_2)$

$\Delta\mathbf{V}_3 = \mathbf{V}_{o3} - \mathbf{V}$
$\dot{\mathbf{z}}_3 = \mathbf{Z}_{s3}(\mathbf{z}_3, \Delta\mathbf{V}_3)$
$\mathbf{F}_3 = \mathbf{Z}_{o3}(\mathbf{z}_3, \Delta\mathbf{V}_3)$



$\dot{\mathbf{V}} = \mathbf{m}^{-1}(\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3)$

Impedance Control Implementation

- Controlling robot impedance is an **ideal**
  - like most control system goals it may be difficult to attain
- How do you control impedance or admittance?
- One primitive (but highly successful) approach:
  - Design low-impedance hardware
    - Low-friction mechanism
      - Kinematic chain of rigid links
    - Effort-controlled actuators
      - e.g., permanent-magnet brushless DC motors
      - high-bandwidth current-controlled amplifiers
  - Use feedback to increase output impedance
    - (Nonlinear) position and velocity feedback control
- This has been called "simple" impedance control
  - (more correctly "simple-minded" impedance control)

Typical Robot Model

- Effort-driven inertia

$$\mathbf{I}(\boldsymbol{\theta})\dot{\boldsymbol{\omega}} + \mathbf{C}(\boldsymbol{\theta},\boldsymbol{\omega}) + \mathbf{G}(\boldsymbol{\theta}) = \boldsymbol{\tau}_{motor} + \boldsymbol{\tau}_{interaction}$$

$\boldsymbol{\theta}$: generalized coordinates, joint angles, configuration variables

$\boldsymbol{\omega}$: generalized velocities, joint angular velocities

$\boldsymbol{\tau}$: generalized forces, joint torques

I: configuration-dependent inertia

C: inertial coupling (Coriolis & centrifugal accelerations)

G: potential forces (gravitational torques)

- Linkage kinematics transform interaction forces to interaction torques

$$\mathbf{X} = \mathbf{L}(\boldsymbol{\theta})$$

$$\mathbf{V} = \dot{\mathbf{X}} = (\partial\mathbf{L}/\partial\boldsymbol{\theta})\dot{\boldsymbol{\theta}} = \mathbf{J}(\boldsymbol{\theta})\boldsymbol{\omega}$$

$$\boldsymbol{\tau}_{interaction} = \mathbf{J}(\boldsymbol{\theta})^t\,\mathbf{F}_{interaction}$$

X: interaction port (end-point) position

V: interaction port (end-point) velocity

$\mathbf{F}_{interaction}$: interaction port force

L: mechanism kinematic equations

J: mechanism Jacobian

Simple Impedance Control

- Target end-point behavior
  - Norton equivalent network with elastic and viscous impedance, possibly nonlinear
- Express as equivalent configuration-space behavior
  - use kinematic transformations
- This defines a position-and-velocity-feedback controller…
  - A ***non-linear*** variant of PD (proportional+derivative) control
- …that will implement the target behavior

$$\mathbf{F}_{impedance} = \mathbf{K}(\mathbf{X}_o - \mathbf{X}) + \mathbf{B}(\mathbf{V}_o - \mathbf{V})$$

$\mathbf{X}_o$: virtual position

$\mathbf{V}_o$: virtual velocity

K: displacement-dependent (elastic) force function

B: velocity-dependent force function

$$\boldsymbol{\tau}_{motor} = \mathbf{J}(\boldsymbol{\theta})^t\,\mathbf{F}_{impedance}$$

$$\boldsymbol{\tau}_{motor} = \mathbf{J}(\boldsymbol{\theta})^t\left(\mathbf{K}(\mathbf{X}_o - \mathbf{L}(\boldsymbol{\theta})) + \mathbf{B}(\mathbf{V}_o - \mathbf{J}(\boldsymbol{\theta})\boldsymbol{\omega})\right)$$

Dynamics of controller impedance coupled to mechanism inertia with interaction port:

$$\mathbf{I}(\boldsymbol{\theta})\dot{\boldsymbol{\omega}} + \mathbf{C}(\boldsymbol{\theta},\boldsymbol{\omega}) + \mathbf{G}(\boldsymbol{\theta}) =$$
$$\mathbf{J}(\boldsymbol{\theta})^t\left(\mathbf{K}(\mathbf{X}_o - \mathbf{L}(\boldsymbol{\theta})) + \mathbf{B}(\mathbf{V}_o - \mathbf{J}(\boldsymbol{\theta})\boldsymbol{\omega})\right)$$
$$+\,\mathbf{J}(\boldsymbol{\theta})^t\,\mathbf{F}_{interaction}$$

Mechanism Singularities

- Impedance control also facilitates interaction with the robot's own mechanics
  - Compare with motion control:
- Position control maps desired end-point trajectory onto configuration space
  - Requires inverse kinematic equations
    - Ill-defined, no general algebraic solution exists
      - one end-point position usually corresponds to many configurations
      - some end-point positions may not be reachable

$$\mathbf{X} = \mathbf{L}(\boldsymbol{\theta})$$

$$\boldsymbol{\theta}_{desired} = \mathbf{L}^{-1}(\mathbf{X}_{desired})$$

- Resolved-rate motion control uses inverse Jacobian
  - Locally linear approach, will find a solution if one exists
  - At some configurations Jacobian becomes singular
    - Motion is not possible in one or more directions

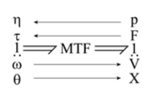
$$\mathbf{V} = \mathbf{J}(\boldsymbol{\theta})\boldsymbol{\omega}$$

$$\boldsymbol{\omega}_{desired} = \mathbf{J}(\boldsymbol{\theta})^{-1}\mathbf{V}_{desired}$$

- A typical motion controller won't work at or near these singular configurations

Mechanism Kinematics

- Mechanism kinematics relate configuration space $\{\boldsymbol{\theta}\}$ to workspace $\{\mathbf{X}\}$
  - In network terms this is like a multi-variable lever*
  - Hence power conjugate variables are well-defined in ***opposite*** directions



*A multiport modulated transformer

- Generalized coordinates uniquely define mechanism configuration
  - *by definition*
- Hence the following maps are ***always*** well-defined
  - generalized coordinates (configuration space) to end-point coordinates (workspace)
  - generalized velocities to workspace velocity
  - workspace force to generalized force
  - workspace momentum to generalized momentum

Control at Mechanism Singularities

- Simple impedance control law was derived by transforming desired behavior…
  - Norton equivalent network in workspace coordinates

  …from workspace to configuration space
- All of the required transformations are ***guaranteed*** well-defined at ***all*** configurations
  - $\mathbf{X} \Leftarrow \boldsymbol{\theta}$
  - $\mathbf{V} \Leftarrow \boldsymbol{\omega}$
  - $\boldsymbol{\tau} \Leftarrow \mathbf{F}$

$$\boldsymbol{\tau}_{motor} = \mathbf{J}(\boldsymbol{\theta})^t\left(\mathbf{K}(\mathbf{X}_o - \mathbf{L}(\boldsymbol{\theta})) + \mathbf{B}(\mathbf{V}_o - \mathbf{J}(\boldsymbol{\theta})\boldsymbol{\omega})\right)$$

- Hence the simple impedance controller can operate ***near, at and through*** mechanism singularities

Generalized Coordinates: A Word of Caution

- Aside:
  - Identification of generalized coordinates requires care
    - Independently variable
    - Uniquely define mechanism configuration
    - Not themselves unique
  - Actuator coordinates are often suitable, but not always
    - Example: Stewart platform
  - Identification of generalized forces also requires care
    - Power conjugates of generalized velocities
    - $P = \boldsymbol{\tau}^t\boldsymbol{\omega}$ or $dW = \boldsymbol{\tau}^t d\boldsymbol{\theta}$
  - Actuator forces are often suitable, not always

Suppose You Need Inverse Kinematics Anyway…

- Generally a tough computational problem
- Modeling & simulation afford simple, effective solutions
  - Assume a simple impedance controller
  - Apply it to a simulated mechanism with simplified dynamics
  - Guaranteed convergence properties
    - Hogan 1984
    - Slotine &Yoerger 1987
- Same approach works for redundant mechanisms
  - Redundant: more generalized coordinates than workspace coordinates
  - Inverse kinematics is fundamentally "ill-posed"
  - Rate control based on Moore-Penrose pseudo-inverse suffers "drift"
  - Proper analysis of effective stiffness eliminates drift
    - Mussa-Ivaldi & Hogan 1991

Hogan, N. (1984) *Some Computational Problems Simplified by Impedance Control*, proc. ASME Conf. on Computers in Engineering, pp. 203-209.

Slotine, J.-J.E., Yoerger, D.R. (1987) *A Rule-Based Inverse Kinematics Algorithm for Redundant Manipulators* Int. J. Robotics & Automation 2(2):86-89

Mussa-Ivaldi, F. A. and Hogan, N. (1991) *Integrable Solutions of Kinematic Redundancy via Impedance Control.* Int. J. Robotics Research, 10(5):481-491

Other Implementations: Intrinsically Variable Impedance

- Feedback control of impedance suffers inevitable imperfections
  - "parasitic" sensor & actuator dynamics
  - communication & computation delays
- Alternative: control impedance using intrinsic properties of the actuators and/or mechanism
  - Variable stiffness, damping, inertia
  - Resonance, anti-resonance
    … etc.
    - Impedance is **NOT** just damped spring-mass behavior

Intrinsically Variable Inertia

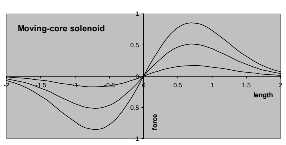
- Inertia is difficult to modulate via feedback but mechanism inertia is a strong function of configuration
- Use excess degrees of freedom to modulate inertia
  - e.g., compare contact with the fist or the fingertips
- Consider the apparent (translational) inertia at the tip of a 3-link open-chain planar mechanism
  - Use mechanism transformation properties
- Translational inertia is usually characterized by $\mathbf{p} = \mathbf{M}\mathbf{v}$
- Generalized (configuration space) inertia is $\boldsymbol{\eta} = \mathbf{I}(\boldsymbol{\theta})\boldsymbol{\omega}$
  - Jacobian: $\mathbf{v} = \mathbf{J}(\boldsymbol{\theta})\boldsymbol{\omega}$, $\boldsymbol{\eta} = \mathbf{J}(\boldsymbol{\theta})^t\mathbf{p}$
  - Corresponding tip (workspace) inertia: $\mathbf{p} = \mathbf{J}(\boldsymbol{\theta})^{-t}\mathbf{I}(\boldsymbol{\theta})\mathbf{J}(\boldsymbol{\theta})^{-1}\mathbf{v}$, $\mathbf{M}_{tip} = \mathbf{J}(\boldsymbol{\theta})^{-t}\mathbf{I}(\boldsymbol{\theta})\mathbf{J}(\boldsymbol{\theta})^{-1}$
- Snag: $\mathbf{J}(\boldsymbol{\theta})$ is not square—inverse $\mathbf{J}(\boldsymbol{\theta})^{-1}$ does not exist

Causal Analysis

- Inertia is an admittance $\quad \mathbf{v} = \mathbf{M}^{-1}\mathbf{p}$
  - prefers integral causality $\quad \boldsymbol{\omega} = \mathbf{I}(\boldsymbol{\theta})^{-1}\boldsymbol{\eta}$
- Transform inverse configuration-space inertia
  - Corresponding tip (workspace) inertia $\quad \mathbf{v} = \mathbf{J}(\boldsymbol{\theta})\mathbf{I}(\boldsymbol{\theta})^{-1}\mathbf{J}(\boldsymbol{\theta})^t\mathbf{p}$
  - This transformation is ***always*** well-defined $\quad \mathbf{M}_{tip}^{-1} = \mathbf{J}(\boldsymbol{\theta})\mathbf{I}(\boldsymbol{\theta})^{-1}\mathbf{J}(\boldsymbol{\theta})^t$
- Does $\mathbf{I}(\boldsymbol{\theta})^{-1}$ always exist?
  - $\mathbf{I}(\boldsymbol{\theta})$ must be symmetric positive definite, hence its inverse exists
- Does $\mathbf{M}_{tip}^{-1}$ always exist?
  - yes, but sometimes it loses rank
    - inverse mass goes to zero in some directions—can't move that way
  - causal argument: input force can always be applied
    - mechanism will "figure out" whether & how to move

Intrinsically Variable Stiffness

- Engineering approaches
  - Moving-core solenoid
  - Variable-pressure air cylinder
  - Pneumatic tension actuator
    - McKibben "muscle"
  - Separately-excited DC machine
    - Fasse et al. 1994
  - …and many more

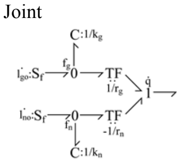


Fasse, E. D., Hogan, N., Gomez, S. R., and Mehta, N. R. (1994) *A Novel Variable Mechanical-Impedance Electromechanical Actuator*. Proc. Symp. Haptic Interfaces for Virtual Environment and Teleoperator Systems, ASME DSC-Vol. 55-1, pp. 311-318.

- Mammalian muscle
  - complex underlying physics
  - stiffness increases with tension
  - antagonist co-contraction increases stiffness (and maybe damping)

Opposing Actuators at a Joint

- Assume
  - constant moment arms
  - linear force-length relation
    - (grossly) simplified model of antagonist muscles about a joint

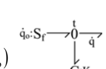


$$l_g = l_{go} - r_g q$$
$$l_n = l_{no} + r_n q$$
$$f_g = k_g l_g$$
$$f_n = k_n l_n$$
$$t = r_g f_g - r_n f_n = r_g k_g (l_{go} - r_g q) - r_n k_n (l_{no} + r_n q)$$
$$t = (r_g k_g l_{go} - r_n k_n l_{no}) - (r_g^2 k_g + r_n^2 k_n) q$$

f: force; l: length; k: actuator stiffness
q: joint angle; t: torque; K: joint stiffness
subscripts: g: agonist; n: antagonist, o: virtual

- Equivalent behavior:
- Opposing torques subtract
- Opposing impedances add
  - Joint stiffness positive if actuator stiffness positive

$$t = K(q_o - q)$$
$$q_o = (r_g k_g l_{go} - r_n k_n l_{no})$$
$$K = (r_g^2 k_g + r_n^2 k_n)$$

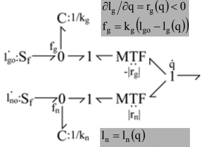


Snag: Configuration-Dependent Moment Arms

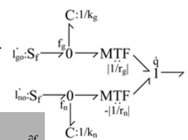
- Connection of linear actuators usually makes moment arm vary with configuration
- Joint stiffness, K:
  - Second term always positive
  - First term may be *negative*

$$l_g = l_g(q)$$
$$\partial l_g / \partial q = r_g(q) < 0$$
$$f_g = k_g\left(l_{go} - l_g(q)\right)$$

$$K = -\left(\frac{\partial r_g}{\partial q} f_g + \frac{\partial r_n}{\partial q} f_n\right) + \left(r_g^2 k_g + r_n^2 k_n\right)$$



More typical: change signs on the transformers

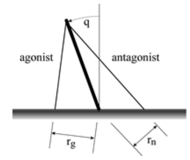


$$l_n = l_n(q)$$
$$\partial l_n / \partial q = r_n(q) > 0$$
$$f_n = k_n\left(l_{no} - l_n(q)\right)$$

$$t = -r_g(q) f_g(q) - r_n(q) f_n(q)$$
$$\frac{\partial t}{\partial q} = -\frac{\partial r_g}{\partial q} f_g - r_g \frac{\partial f_g}{\partial q} - \frac{\partial r_n}{\partial q} f_n - r_n \frac{\partial f_n}{\partial q}$$

This is the "Tent-pole" Effect

- Consequences of configuration-dependent moment arms:
- Opposing "ideal" (zero-impedance) tension actuators
  - agonist moment grows with angle, antagonist moment declines
  - ***always*** unstable
- Constant-stiffness actuators
  - stable only for limited tension
- Mammalian muscle:
  - stiffness is proportional to tension
  - good approximation of complex behavior
  - ***can be stable for all tension***



- Take-home messages:
- Kinematics matters
  - "Kinematic" stiffness may dominate
- Impedance matters
  - Zero output impedance may be highly undesirable

Contact and Coupled Instability

- A GENERAL Problem:
  - Contact and interaction with objects couples their dynamics into the manipulator control system
  - This change may cause instability
    - Example:
      - integral-action motion controller
      - coupling to more mass evokes instability
  - Impedance control affords a solution:
    - Make the manipulator impedance behave like a passive physical system

Hogan, N. (1988) *On the Stability of Manipulators Performing Contact Tasks*, IEEE Journal of Robotics and Automation, 4: 677-686.

Example: Integral-Action Motion Controller

- System:
  - Mass restrained by linear spring & damper, driven by control actuator & external force

$$(ms^2 + bs + k)\,x = cu - f$$
$$\frac{x}{u} = \frac{c}{ms^2 + bs + k}$$

- Controller:
  - Integral of trajectory error

$$u = \frac{g}{s}(r - x)$$

- System + controller:

$$(ms^3 + bs^2 + ks + cg)\,x = cgr - s\,f$$
$$\frac{x}{r} = \frac{cg}{ms^3 + bs^2 + ks + cg}$$

- Isolated stability:
  - Stability requires upper bound on controller gain

$$\frac{bk}{cm} > g$$

s: Laplace variable
x: displacement variable
f: external force variable
u: control input variable
r: reference input variable
m: mass constant
b: damping constant
k: stiffness constant
c: actuator force constant
g: controller gain constant

Example (continued)

- Object mass: $f = m_e s^2\, x$ — m_e: object mass constant
- Coupled system: $[(m + m_e)s^3 + bs^2 + ks + cg]\,x = cgr$

$$\frac{x}{r} = \frac{cg}{(m + m_e)s^3 + bs^2 + ks + cg}$$

- Coupled stability: $bk > cg(m + m_e)$
- Choose ***any*** positive controller gain that will ensure isolated stability: $\frac{bk}{cm} > g$
- That controlled system is ***destabilized*** by coupling to a sufficiently large mass: $m_e > \frac{bk}{cg} - m$

Problem & Approach

- Problem:
  - Find conditions to avoid instability due to contact & interaction
- Approach:
  - Describe the manipulator and its controller as an ***equivalent physical system***
  - Find an (equivalent) physical behavior that will avoid contact/coupled instability
    - Use our knowledge of physical system behavior and how it is constrained
  - Design the controller to impose that desired interaction-port behavior

General Object Dynamics

- Assume:
  - Lagrangian dynamics
  - Passive
  - Neutrally stable in isolation
- Legendre transform:
  - Kinetic co-energy to kinetic energy
  - Lagrangian form to Hamiltonian form
- Hamiltonian = total system energy

$$H_e(\mathbf{p}_e,\mathbf{q}_e)=E_k(\mathbf{p}_e,\mathbf{q}_e)+E_p(\mathbf{q}_e)$$

$$L(\mathbf{q}_e,\dot{\mathbf{q}}_e)=E_k^*(\mathbf{q}_e,\dot{\mathbf{q}}_e)-E_p(\mathbf{q}_e)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\mathbf{q}}_e}\right)-\frac{\partial L}{\partial \mathbf{q}_e}=\mathbf{P}_e-\mathbf{D}_e(\mathbf{q}_e,\dot{\mathbf{q}}_e)$$

$$\mathbf{p}_e=\partial L/\partial\dot{\mathbf{q}}_e=\partial E_k^*/\partial\dot{\mathbf{q}}_e$$

$$E_k(\mathbf{p}_e,\mathbf{q}_e)=\mathbf{p}_e^t\dot{\mathbf{q}}_e-E_k^*(\mathbf{q}_e,\dot{\mathbf{q}}_e)$$

$$H_e(\mathbf{p}_e,\mathbf{q}_e)=\mathbf{p}_e^t\dot{\mathbf{q}}_e-L(\mathbf{q}_e,\dot{\mathbf{q}}_e)$$

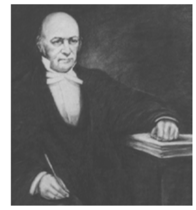
$$\dot{\mathbf{q}}_e=\partial H_e/\partial\mathbf{p}_e$$

$$\dot{\mathbf{p}}_e=-\partial H_e/\partial\mathbf{q}_e-\mathbf{D}_e+\mathbf{P}_e$$

$\mathbf{q}_e$: (generalized) coordinates
L: Lagrangian
E_k^*: kinetic co-energy
E_p: potential energy
$\mathbf{D}_e$: dissipative (generalized) forces
$\mathbf{P}_e$: exogenous (generalized) forces
H_e: Hamiltonian

Sir William Rowan Hamilton

- William Rowan Hamilton
  - Born 1805, Dublin, Ireland
  - Knighted 1835
  - First Foreign Associate elected to U.S. National Academy of Sciences
  - Died 1865
- Accomplishments
  - Optics
  - Dynamics
  - Quaternions
  - Linear operators
  - Graph theory
  - …and more
    - http://www.maths.tcd.ie/pub/HistMath/People/Hamilton/



Aside: some Irishmen of note ☺

- Bishop George **Berkeley** (if a tree falls in the forest …)
- Robert **Boyle** (Boyle's law ...)
- John Boyd **Dunlop**
- George Francis **Fitzgerald** (Lorentz-Fitzgerald contraction)
- William Rowan **Hamilton**
- William Thomson (Lord **Kelvin**)
- Joseph **Larmor**
- Charles **Parsons**
- Osborne **Reynolds**
- George Gabriel **Stokes**
- William Sealy **Gossett** (Student of the t-test)
- Frank **Wilcoxon** (Rank-sum test)

Passivity

- Basic idea: system cannot supply power indefinitely
  - Many alternative definitions, the best are energy-based
    - Wyatt et al. (1981)
- Passive: total system energy is lower-bounded
  - More precisely, *available* energy is lower-bounded
- Power flux may be positive or negative
  - Convention: power positive in
  - Power in (positive)—no limit
  - Power out (negative)—only until stored energy exhausted
    - You can store as much energy as you want but you can withdraw only what was initially stored (a finite amount)
- Passivity ≠ stability
  - Example:
    - Interaction between similarly charged beads, one fixed, one free to move on a wire

Wyatt, J. L., Chua, L. O., Gannett, J. W., Göknar, I. C. and Green, D. N. (1981) Energy Concepts in the State-Space Theory of Nonlinear n-Ports: Part I — Passivity. IEEE Transactions on Circuits and Systems, Vol. CAS-28, No. 1, pp. 48-61.

Stability

- Stability:
  - In the sense of convergence to equilibrium
- Use Lyapunov's second method
  - A generalization of energy-based analysis
  - Lyapunov function: positive-definite non-decreasing state function
  - Sufficient condition for asymptotic stability: Negative semi-definitive rate of change of Lyapunov function
- For physical systems total energy ***may*** be a useful candidate Lyapunov function
  - Equilibria are at an energy minima
  - Dissipation ⇒ energy reduction ⇒ convergence to equilibrium
  - Hamiltonian form describes dynamics in terms of total energy

Steady State & Equilibrium

- Steady state:
  - Kinetic energy is a positive-definite non-decreasing function of generalized momentum
- Assume:
  - Dissipative (internal) forces vanish in steady-state
    - Rules out static (Coulomb) friction
  - Potential energy is a positive-definite non-decreasing function of generalized displacement
    - Steady-state is a unique equilibrium configuration
- Steady state is equilibrium at the origin of the state space $\{\mathbf{p}_e,\mathbf{q}_e\}$

$$\dot{\mathbf{q}}_e=\mathbf{0}=\partial H_e/\partial\mathbf{p}_e=\partial E_k/\partial\mathbf{p}_e$$

$$\partial E_k/\partial\mathbf{p}_e=\mathbf{0}\Rightarrow\mathbf{p}_e=\mathbf{0}$$

$$\dot{\mathbf{p}}_e=\mathbf{0}=-\partial H_e/\partial\mathbf{q}_e-\mathbf{D}_e+\mathbf{P}_e$$

$$\text{Assume }\mathbf{D}_e(\mathbf{0},\mathbf{q}_e)=\mathbf{0}$$

$$\text{Isolated}\Rightarrow\mathbf{P}_e=\mathbf{0}$$

$$\left.\frac{\partial H_e}{\partial\mathbf{q}_e}\right|_{\mathbf{p}_e=\mathbf{0}}=\left.\frac{\partial E_k}{\partial\mathbf{q}_e}\right|_{\mathbf{p}_e=\mathbf{0}}+\frac{\partial E_p}{\partial\mathbf{q}_e}$$

$$\left.\frac{\partial E_k}{\partial\mathbf{q}_e}\right|_{\mathbf{p}_e=\mathbf{0}}=\mathbf{0}\qquad\therefore\left.\frac{\partial H_e}{\partial\mathbf{q}_e}\right|_{\mathbf{p}_e=\mathbf{0}}=\frac{\partial E_p}{\partial\mathbf{q}_e}$$

$$\partial E_p/\partial\mathbf{q}_e=\mathbf{0}\Rightarrow\mathbf{q}_e=\mathbf{0}$$

Notation

- Represent partial derivatives using subscripts
- H_e is a scalar
  - the Hamiltonian state function
- $\mathbf{H}_{eq}$ is a vector
  - Partial derivatives of the Hamiltonian w.r.t. each element of $\mathbf{q}_e$
- $\mathbf{H}_{ep}$ is a vector
  - Partial derivatives of the Hamiltonian w.r.t. each element of $\mathbf{p}_e$

$$\mathbf{H}_{eq} = \frac{\partial H_e}{\partial \mathbf{q}_e}$$

$$\mathbf{H}_{ep} = \frac{\partial H_e}{\partial \mathbf{p}_e}$$

$$\dot{\mathbf{q}}_e = \mathbf{H}_{ep}(\mathbf{p}_e, \mathbf{q}_e)$$

$$\dot{\mathbf{p}}_e = -\mathbf{H}_{eq}(\mathbf{p}_e, \mathbf{q}_e) - \mathbf{D}_e(\mathbf{p}_e, \mathbf{q}_e) + \mathbf{P}_e$$

Isolated Stability

- Use the Hamiltonian as a Lyapunov function
  - Positive-definite non-decreasing function of state
  - Rate of change of stored energy = power in – power dissipated
- Sufficient condition for asymptotic stability:
  - Dissipative generalized forces are a positive-definite function of generalized momentum
    - Dissipation may vanish if $\mathbf{p}_e = \mathbf{0}$ and system is not at equilibrium
    - But $\mathbf{p}_e = \mathbf{0}$ does not describe any system trajectory
      - LaSalle-Lefshetz theorem
  - Energy decreases on all non-equilibrium system trajectories

$$dH_e/dt = \mathbf{H}_{eq}^t \dot{\mathbf{q}}_e + \mathbf{H}_{ep}^t \dot{\mathbf{p}}_e$$

$$dH_e/dt = \mathbf{H}_{eq}^t \mathbf{H}_{ep} + \mathbf{H}_{ep}^t(-\mathbf{H}_{eq} - \mathbf{D}_e + \mathbf{P}_e)$$

$$dH_e/dt = \dot{\mathbf{q}}_e^t \mathbf{P}_e - \dot{\mathbf{q}}_e^t \mathbf{D}_e$$

$$\text{Isolated} \Rightarrow \mathbf{P}_e = \mathbf{0}$$

$$\therefore dH_e/dt = -\dot{\mathbf{q}}_e^t \mathbf{D}_e$$

$$\dot{\mathbf{q}}_e^t \mathbf{D}_e > 0 \Rightarrow dH_e/dt < 0 \quad \forall\, \mathbf{p}_e \neq \mathbf{0}$$

Physical System Interaction

- Interaction of general dynamic systems
  - Many possibilities: cascade, parallel, feedback…
    - Two linear systems: $y_1 = G_1(s)u_1$, $y_2 = G_2(s)u_2$
    - Cascade coupling equations: $y_3 = y_2$, $u_2 = y_1$, $u_1 = u_3$
    - Combination: $y_3 = G_3(s)u_3$, $G_3(s) = G_2(s)G_1(s)$
- ***Not*** power-continuous

$$y_3 u_3 \neq y_2 u_2 + y_1 u_1$$

- Interaction of physical systems
  - If u_i and y_i are power conjugates
  - G_i are impedances or admittances
  - Connection must be power-continuous:
    - Power into coupled system must equal net power into component systems

$$u_3 y_3 = u_1 y_1 + u_2 y_2$$

- Physical systems **cannot be cascade connected** at an interaction port

Parallel & Feedback Connections

- Power continuity $\quad y_3 u_3 = y_2 u_2 + y_1 u_1$
- Parallel connection equations $\quad y_3 = \pm y_2 \pm y_1$, $u_3 = u_2 = u_1$
- Power balance $\quad y_3 u_3 = \pm y_2 u_2 \pm y_1 u_1$
  —OK
- Feedback connection equations $\quad y_3 = y_1 = u_2$, $u_1 = u_3 - y_2$
- Power balance $\quad u_1 y_1 = u_3 y_3 - y_2 u_2$
  —OK

Interaction Port

- Assume coupling occurs at a set of points on the object $\mathbf{X}_e$
  - Let this define an interaction port
  - $\mathbf{X}_e$ is as a function of generalized coordinates $\mathbf{q}_e$ $\quad \mathbf{X}_e = \mathbf{L}_e(\mathbf{q}_e)$
  - Generalized velocity determines port velocity $\quad \mathbf{V}_e = \mathbf{J}_e(\mathbf{q}_e)\dot{\mathbf{q}}_e$
  - Port force determines generalized force $\quad \mathbf{P}_e = \mathbf{J}_e^t(\mathbf{q}_e)\mathbf{F}_e$
- These relations are always well-defined
  - Guaranteed by the definition of generalized coordinates

This ensures power continuity

Simple Impedance

- Target (**ideal**) behavior of manipulator $\quad \mathbf{F}_z = \mathbf{K}(\mathbf{X}_z - \mathbf{X}_o) + \mathbf{B}(\mathbf{V}_z)$
  - Elastic and viscous behavior
- In Hamiltonian form:
  - Hamiltonian = potential energy
  - Assume $\mathbf{V}_o = \mathbf{0}$ for stability analysis
  - Isolated: $\mathbf{V}_z = \mathbf{0}$ or $\mathbf{F}_z = \mathbf{0}$
  - Sufficient condition for isolated asymptotic stability:

$$\dot{\mathbf{p}}_z = \mathbf{H}_{zq}(\mathbf{q}_z) + \mathbf{B}(\mathbf{V}_z) \qquad \mathbf{q}_z = \mathbf{X}_z - \mathbf{X}_o$$

$$\dot{\mathbf{q}}_z = \mathbf{V}_z - \mathbf{V}_o \qquad H_z(\mathbf{q}_z) = \int \mathbf{K}(\mathbf{q}_z)d\mathbf{q}_z$$

$$\mathbf{F}_z = \dot{\mathbf{p}}_z$$

$$\mathbf{V}_o = \mathbf{V}_z = \mathbf{0} \Rightarrow \mathbf{q}_z = \text{constant} \Rightarrow \mathbf{F}_z = \text{constant}$$

$$\mathbf{F}_z = \mathbf{0} \Rightarrow \mathbf{H}_{zq} = -\mathbf{B} \therefore dH_z/dt = \mathbf{H}_{zq}^t \dot{\mathbf{q}}_z = -\mathbf{B}^t \dot{\mathbf{q}}_z$$

$$\mathbf{B}^t \dot{\mathbf{q}}_z > 0 \quad \forall \mathbf{V}_z \neq \mathbf{0}$$

- Unconstrained mass in Hamiltonian form
  - Hamiltonian = kinetic energy
  - Arbitrarily small mass

$$\dot{\mathbf{q}}_e = \mathbf{H}_{ep}(\mathbf{p}_e) \qquad H_e(\mathbf{p}_e) = \tfrac{1}{2}\mathbf{p}_e^t \mathbf{M}^{-1} \mathbf{p}_e$$

$$\dot{\mathbf{p}}_e = \mathbf{F}_e$$

$$\mathbf{V}_e = \dot{\mathbf{q}}_e$$

- Couple these with common velocity

$$\mathbf{V}_e = \mathbf{V}_z$$

$$\mathbf{F}_e^t \mathbf{V}_e + \mathbf{F}_z^t \mathbf{V}_z = \mathbf{0}$$

Mass Coupled to Simple Impedance

- Hamiltonian form
  - Total energy = sum of components
- Assume positive-definite, non-decreasing potential energy
  - Equilibrium at $(\mathbf{p}_e,\mathbf{q}_z) = (\mathbf{0},\mathbf{0})$
- Rate of change of Hamiltonian:
- Sufficient condition for asymptotic stability
  - And because mass is unconstrained, stability is global

$$\mathrm{H}_t(\mathbf{p}_e,\mathbf{q}_z) = \mathrm{H}_e(\mathbf{p}_e) + \mathrm{H}_z(\mathbf{q}_z)$$
$$\dot{\mathbf{p}}_e = -\mathbf{H}_{tq}(\mathbf{q}_z) - \mathbf{B}(\mathbf{H}_{tp}(\mathbf{p}_e))$$
$$\dot{\mathbf{q}}_z = \mathbf{H}_{tp}(\mathbf{p}_e)$$
$$d\mathrm{H}_t/dt = \mathbf{H}_{tp}^t\dot{\mathbf{p}}_e + \mathbf{H}_{tq}^t\dot{\mathbf{q}}_z$$
$$d\mathrm{H}_t/dt = -\mathbf{H}_{tp}^t\mathbf{H}_{tq} - \mathbf{H}_{tp}^t\mathbf{B} + \mathbf{H}_{tq}^t\mathbf{H}_{tp} = -\dot{\mathbf{q}}_z^t\mathbf{B}$$
$$\dot{\mathbf{q}}_z^t\mathbf{B} > 0 \quad \forall \mathbf{p}_e \neq \mathbf{0}$$

General Object Coupled to Simple Impedance

- Total Hamiltonian (energy) is sum of components
- Assume
  - Both systems at equilibrium
  - Interaction port positions coincide at coupling
- Total energy is a positive-definite, non-decreasing state function
- Rate of change of energy:

$$\mathrm{H}_t(\mathbf{p}_e,\mathbf{q}_e) = \mathrm{H}_e(\mathbf{p}_e,\mathbf{q}_e) + \mathrm{H}_z(\mathbf{q}_z)$$
$$\mathrm{H}_t(\mathbf{p}_e,\mathbf{q}_e) = \mathrm{E}_k(\mathbf{p}_e,\mathbf{q}_e) + \mathrm{E}_p(\mathbf{q}_e) + \mathrm{H}_z(\mathbf{L}_e(\mathbf{q}_e) - \mathbf{X}_o)$$
$$d\mathrm{H}_t/dt = \mathbf{H}_{zq}^t\mathbf{J}_e\mathbf{H}_{ep} + \mathbf{H}_{eq}^t\mathbf{H}_{ep} - \mathbf{H}_{ep}^t\mathbf{H}_{eq}$$
$$-\mathbf{H}_{ep}^t\mathbf{D}_e - \mathbf{H}_{ep}^t\mathbf{J}_e^t\mathbf{H}_{zq} - \mathbf{H}_{ep}^t\mathbf{J}_e^t\mathbf{B}$$
$$d\mathrm{H}_t/dt = -\dot{\mathbf{q}}_e^t\mathbf{D}_e - \dot{\mathbf{q}}_z^t\mathbf{B}$$

- The previous conditions sufficient for stability of
  - Object in isolation
  - Simple impedance coupled to arbitrarily small mass
- …ensure global asymptotic coupled stability
  - Energy decreases on all non-equilibrium state trajectories
  - **True for objects of arbitrary dynamic order**

Simple Impedance Controller Implementation

- Robot model:
  - Inertial mechanism, statically balanced (or zero gravity), effort-controlled actuators
    - Hamiltonian = kinetic energy
- Controller:
  - Transform simple impedance to manipulator configuration space
- Controller coupled to robot:
  - Same structure as a physical system with Hamiltonian H_c

$$\mathrm{H}_c = \mathrm{H}_m + \mathrm{H}_z$$

$$\dot{\mathbf{q}}_m = \mathbf{H}_{mp} \qquad \mathrm{H}_m = \tfrac{1}{2}\mathbf{p}_m^t\mathbf{I}^{-1}(\mathbf{q}_m)\mathbf{p}_m$$
$$\dot{\mathbf{p}}_m = -\mathbf{H}_{mq} - \mathbf{D}_m + \mathbf{P}_a + \mathbf{J}_m^t\mathbf{F}_m$$
$$\mathbf{V}_m = \mathbf{J}_m\dot{\mathbf{q}}_m$$
$$\mathbf{X}_m = \mathbf{L}_m(\mathbf{q}_m)$$
$$\mathbf{P}_a = -\mathbf{J}_m^t\{\mathbf{K}(\mathbf{L}_m(\mathbf{q}_m) - \mathbf{X}_o) - \mathbf{B}(\mathbf{J}_m\dot{\mathbf{q}}_m)\}$$
$$\dot{\mathbf{q}}_m = \mathbf{H}_{cp}$$
$$\dot{\mathbf{p}}_m = -\mathbf{H}_{cq} - \mathbf{D}_m - \mathbf{J}_m^t\mathbf{B} + \mathbf{J}_m^t\mathbf{F}_m$$
$$\mathbf{V}_m = \mathbf{J}_m\dot{\mathbf{q}}_m$$
$$\mathbf{X}_m = \mathbf{L}_m(\mathbf{q}_m)$$

$\mathbf{q}_m$: generalized coordinates (configuration variables)
$\mathbf{p}_m$: generalized momenta
H_m: Hamiltonian
$\mathbf{I}$: inertia
$\mathbf{D}_m$: dissipative (generalized) forces
$\mathbf{P}_a$: actuator (generalized) forces
$\mathbf{X}_m,\mathbf{V}_m,\mathbf{F}_m$: interaction port position, velocity, force
$\mathbf{L}_m,\mathbf{J}_m$: kinematic equations, Jacobian

Simple Impedance Controller Isolated Stability

- Rate of change of Hamiltonian:
- Energy decreases on all non-equilibrium trajectories if
  - System is isolated $\mathbf{F}_m = \mathbf{0}$
  - Dissipative forces are positive-definite $\dot{\mathbf{q}}_m^t\mathbf{D}_m > 0, \mathbf{V}_m^t\mathbf{B} > 0 \quad \forall \mathbf{p}_m \neq \mathbf{0}$

$$d\mathrm{H}_c/dt = \mathbf{H}_{cq}^t\mathbf{H}_{cp} - \mathbf{H}_{cp}^t\mathbf{H}_{cq} - \mathbf{H}_{cp}^t\mathbf{D}_m$$
$$-\mathbf{H}_{cp}^t\mathbf{J}_m^t\mathbf{B} + \mathbf{H}_{cp}^t\mathbf{J}_m^t\mathrm{F}_m$$
$$d\mathrm{H}_c/dt = -\dot{\mathbf{q}}_m^t\mathbf{D}_m - \mathbf{V}_m^t\mathbf{B} + \mathbf{V}_m^t\mathbf{F}_m$$
$$\mathbf{F}_m = \mathbf{0} \Rightarrow d\mathrm{H}_c/dt = -\dot{\mathbf{q}}_m^t\mathbf{D}_m - \mathbf{V}_m^t\mathbf{B}$$

- Minimum energy is at $\mathbf{q}_z = \mathbf{0}, \mathbf{X}_m = \mathbf{X}_o$
  - But this may not define a unique manipulator configuration
  - Hamiltonian is a positive-definite non-decreasing function of $\mathbf{q}_z$ but usually *not* of configuration $\mathbf{q}_m$
- Interaction-port impedance may not control internal degrees of freedom
  - Could add terms to controller but for simplicity…
- Assume:
  - Non-redundant mechanism
  - Non-singular Jacobian
- Then
  - Hamiltonian is positive-definite & non-decreasing in a region about $\mathbf{q}_m = \mathbf{L}^{-1}(\mathbf{X}_o)$
- *Local* asymptotic stability

Simple Impedance Controller Coupled Stability

- Coupling kinematics $\mathbf{q}_t = \mathbf{q}_t(\mathbf{q}_m,\mathbf{q}_e)$
  - Coupling relates $\mathbf{q}_m$ to $\mathbf{q}_e$ but no need to solve explicitly
  - Total Hamiltonian (energy) is sum of components $\mathrm{H}_t = \mathrm{H}_e(\mathbf{p}_e,\mathbf{q}_e) + \mathrm{H}_c(\mathbf{p}_m,\mathbf{q}_m)$
- Rate of change of Hamiltonian

$$d\mathrm{H}_t/dt = \mathbf{H}_{eq}^t\mathbf{H}_{ep} + \mathbf{H}_{ep}^t(-\mathbf{H}_{eq} - \mathbf{D}_e + \mathbf{J}_e^t\mathbf{F}_e)$$
$$+\mathbf{H}_{cq}^t\mathbf{H}_{cp} + \mathbf{H}_{cp}^t(-\mathbf{H}_{cq} - \mathbf{D}_m - \mathbf{J}_m^t\mathbf{B} + \mathbf{J}_m^t\mathbf{F}_m)$$
$$d\mathrm{H}_t/dt = -\dot{\mathbf{q}}_e^t\mathbf{D}_e + \dot{\mathbf{q}}_e^t\mathbf{J}_e^t\mathbf{F}_e - \dot{\mathbf{q}}_m^t(\mathbf{D}_m + \mathbf{J}_m^t\mathbf{B}) + \dot{\mathbf{q}}_m^t\mathbf{J}_m^t\mathbf{F}_m$$
$$d\mathrm{H}_t/dt = -\dot{\mathbf{q}}_e^t\mathbf{D}_e + \mathbf{V}_e^t\mathbf{F}_e - \dot{\mathbf{q}}_m^t\mathbf{D}_m - \mathbf{V}_m^t\mathbf{B} + \mathbf{V}_m^t\mathbf{F}_m$$

- Coupling cannot generate power

$$\mathbf{V}_e^t\mathbf{F}_e + \mathbf{V}_m^t\mathbf{F}_m = 0$$
$$\therefore d\mathrm{H}_t/dt = -\dot{\mathbf{q}}_e^t\mathbf{D}_e - \dot{\mathbf{q}}_m^t\mathbf{D}_m - \mathbf{V}_m^t\mathbf{B}$$

- The previous conditions sufficient for stability of
  - Object in isolation
  - Simple impedance controlled robot
- …ensure local asymptotic coupled stability

Kinematic Errors

- Assume controller and interaction port kinematics differ
  - Controller kinematics maps configuration to a point $\tilde{\mathbf{X}}$
  - Corresponding potential function is positive-definite, non-decreasing in a region about $\tilde{\mathbf{q}}_m = \tilde{\mathbf{L}}^{-1}(\mathbf{X}_o)$
- Assume self-consistent controller kinematics
  - The (erroneous) Jacobian is the correct derivative of the (erroneous) kinematics
  - e.g., contact doesn't occur where you planned

$$\mathbf{P}_a = -\tilde{\mathbf{J}}^t\{\mathbf{K}(\tilde{\mathbf{L}}(\mathbf{q}_m) - \mathbf{X}_o) - \mathbf{B}(\tilde{\mathbf{J}}\dot{\mathbf{q}}_m)\}$$
$$\tilde{\mathbf{X}} = \tilde{\mathbf{L}}(\mathbf{q}_m) \neq \mathbf{L}_m(\mathbf{q}_m)$$
$$\tilde{\mathrm{H}}_z(\mathbf{q}_m) = \mathrm{H}_z(\tilde{\mathbf{q}}_z) = \mathrm{H}_z(\tilde{\mathbf{L}}(\mathbf{q}_m) - \mathbf{X}_o)$$
$$\partial\tilde{\mathbf{L}}/\partial\mathbf{q}_m = \tilde{\mathbf{J}}$$
$$d\tilde{\mathbf{X}}/dt = \tilde{\mathbf{V}} = \tilde{\mathbf{J}}(\mathbf{q}_m)\dot{\mathbf{q}}_m$$
$$d\tilde{\mathrm{H}}_z/dt = \mathbf{H}_{zq}^t\frac{\partial\tilde{\mathbf{L}}}{\partial\mathbf{q}_m}\dot{\mathbf{q}}_m = \mathbf{H}_{zq}^t\tilde{\mathbf{J}}\dot{\mathbf{q}}_m = \mathbf{H}_{zq}^t\tilde{\mathbf{V}}$$

Kinematic Errors (continued)

- Hamiltonian of this controller coupled to the robot
  - Hamiltonian state equations
  - Rate of change of the Hamiltonian
  - In isolation
- Previous conditions on $\mathbf{D}_m$ & $\mathbf{B}$ are sufficient for isolated local asymptotic stability

$$\tilde{H}_c(\mathbf{p}_m,\mathbf{q}_m) = H_m(\mathbf{p}_m,\mathbf{q}_m) + H_z(\tilde{\mathbf{q}}_z)$$

$$\tilde{H}_c(\mathbf{p}_m,\mathbf{q}_m) = H_m(\mathbf{p}_m,\mathbf{q}_m) + H_z(\tilde{\mathbf{L}}(\mathbf{q}_m) - \mathbf{X}_o)$$

$$\dot{\mathbf{q}}_m = \mathbf{H}_{mp}$$

$$\dot{\mathbf{p}}_m = -\mathbf{H}_{mq} - \mathbf{D}_m - \tilde{\mathbf{J}}^t\mathbf{H}_{zq} - \tilde{\mathbf{J}}^t\mathbf{B} + \mathbf{J}_m^t\mathbf{F}_m$$

$$d\tilde{H}_c/dt = \mathbf{H}_{zq}^t\tilde{\mathbf{J}}\mathbf{H}_{mp} + \mathbf{H}_{mq}^t\mathbf{H}_{mp} + \mathbf{H}_{mp}^t\left(-\mathbf{H}_{mq} - \mathbf{D}_m - \tilde{\mathbf{J}}^t\mathbf{H}_{zq} - \tilde{\mathbf{J}}^t\mathbf{B} + \mathbf{J}_m^t\mathbf{F}_m\right)$$

$$d\tilde{H}_c/dt = -\dot{\mathbf{q}}_m^t\mathbf{D}_m - \tilde{\mathbf{V}}^t\mathbf{B} + \mathbf{J}_m^t\mathbf{F}_m$$

$$\mathbf{F}_m = \mathbf{0} \Rightarrow d\tilde{H}_c/dt = -\dot{\mathbf{q}}_m^t\mathbf{D}_m - \tilde{\mathbf{V}}^t\mathbf{B}$$

Insensitivity to Kinematic Errors

- The same conditions are also sufficient to ensure local asymptotic coupled stability
  - Coupled system Hamiltonian and its rate of change:
- Stability properties are insensitive to kinematic errors
  - Provided they are self-consistent
- Note that these results ***do not require small kinematic errors***
  - Could arise when contact occurs at unexpected locations
  - e.g., on the robot links rather than the end-point

$$\tilde{H}_t = E_k(\mathbf{p}_e,\mathbf{q}_e) + E_p(\mathbf{q}_e) + H_m(\mathbf{p}_m,\mathbf{q}_m) + H_z(\tilde{\mathbf{L}}(\mathbf{q}_m) - \mathbf{X}_o)$$

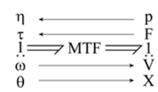
$$d\tilde{H}_t/dt = -\dot{\mathbf{q}}_e^t\mathbf{D}_e - \dot{\mathbf{q}}_m^t\mathbf{D}_m - \tilde{\mathbf{V}}^t\mathbf{B}$$

Remarks

- Interaction stability
  - The above results can be extended
    - Neutrally stable objects
    - Kinematic constraints
      - no dynamics
    - Interface dynamics
      - e.g., due to sensors
  - "Simple" impedance control can provide a robust solution to the contact instability problem
    - But it depends heavily on ideal effort-controlled actuators
- Structure matters
  - Dynamics of physical systems are constrained in useful ways
- It may be beneficial to ***impose*** physical system structure on a general dynamic system
  - e.g. a robot controller
- That's the main idea underlying impedance control

Apparent Mechanical Behavior

- *Apparent* behavior matters
  - —what something feels like where you touch it.
  - Contact coordinates are usually different from generalized coordinates
    - e.g. robot joint angles vs. end-point coordinates
- Mechanical physics constrains how behavior transforms
  - Changing coordinates affects physical variables differently
    - Position, velocity, force, momentum, (etc.) transform differently
  - Conjugate variables are ***uniquely*** defined in ***opposite directions***
    - Generalized coordinates uniquely determine contact coordinates
    - Contact forces uniquely define generalized forces



  - **Redundancy doesn't matter**
    - True with more generalized coordinates than contact coordinates
- A consequence of **bi-lateral interaction** and power continuity

Simple Impedance Controller

- As much design as control
  - Highly-backdrivable mechanics
    - Current-controlled motors
    - No or minimal gearing
    - Very low friction
    - Inertia-dominated dynamics
  - Specify desired end-point behavior
    - ***Nonlinear*** spring & damper
  - Transform to actuator coordinates
    - Transformations are guaranteed well-defined
- **Crude—but effective!**
  - Doesn't compensate for inertia or friction but …
  - **Can operate at "singularities"**
  - **Strongly robust coupled stability**

$$\mathbf{I}(\boldsymbol{\theta})\dot{\boldsymbol{\omega}} + \mathbf{C}(\boldsymbol{\theta},\boldsymbol{\omega}) = \boldsymbol{\tau}_{motor} + \boldsymbol{\tau}_{interaction}$$

$$\mathbf{F}_{impedance} = \mathbf{K}(\mathbf{X}_o - \mathbf{X}) + \mathbf{B}(\mathbf{V}_o - \mathbf{V})$$

$$\mathbf{X} = \mathbf{L}(\boldsymbol{\theta})$$

$$\mathbf{V} = \dot{\mathbf{X}} = (\partial\mathbf{L}/\partial\boldsymbol{\theta})\dot{\boldsymbol{\theta}} = \mathbf{J}(\boldsymbol{\theta})\boldsymbol{\omega}$$

$$\boldsymbol{\tau}_{interaction} = \mathbf{J}(\boldsymbol{\theta})^t\mathbf{F}_{interaction}$$

$$\boldsymbol{\tau}_{motor} = \mathbf{J}(\boldsymbol{\theta})^t\mathbf{F}_{impedance}$$

$$\boldsymbol{\tau}_{motor} = \mathbf{J}(\boldsymbol{\theta})^t\left(\mathbf{K}(\mathbf{X}_o - \mathbf{L}(\boldsymbol{\theta})) + \mathbf{B}(\mathbf{V}_o - \mathbf{J}(\boldsymbol{\theta})\boldsymbol{\omega})\right)$$

$$\mathbf{I}(\boldsymbol{\theta})\dot{\boldsymbol{\omega}} + \mathbf{C}(\boldsymbol{\theta},\boldsymbol{\omega}) = \mathbf{J}(\boldsymbol{\theta})^t\mathbf{F}_{interaction} + \mathbf{J}(\boldsymbol{\theta})^t\left(\mathbf{K}(\mathbf{X}_o - \mathbf{L}(\boldsymbol{\theta})) + \mathbf{B}(\mathbf{V}_o - \mathbf{J}(\boldsymbol{\theta})\boldsymbol{\omega})\right)$$

$\boldsymbol{\theta}$: generalized coordinates	$\mathbf{F}_{interaction}$: interaction port force
$\boldsymbol{\omega}$: generalized velocities	$\mathbf{L}$: mechanism kinematics
$\boldsymbol{\tau}$: generalized forces	$\mathbf{J}$: mechanism Jacobian
$\mathbf{I}$: configuration-dependent inertia	$\mathbf{X}_o$: virtual position
$\mathbf{C}$: Coriolis & centrifugal terms	$\mathbf{V}_o$: virtual velocity
$\mathbf{X}$: interaction port position	$\mathbf{K}$: displacement-dependent force
$\mathbf{V}$: interaction port velocity	$\mathbf{B}$: velocity-dependent force

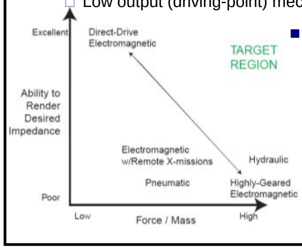
Robot-Mediated Therapy

- **Contact and interaction are essential**
- More than twice the benefit of conventional therapy alone
- Fewer side effects (joint pain)

Hogan, N., Krebs, H.I., Rohrer, B., Palazzolo, J.J., Dipietro, L., Fasoli, S.E., Stein, J., Frontera, W.R., Volpe, B.T., (2006) *Motions or Muscles? Some Behavioral Factors Underlying Robotic Assistance of Motor Recovery.* VA Journal of Rehabilitation Research and Development, 43(5):605-618.

Contact Robotics Requires High Force & Low, Variable Impedance

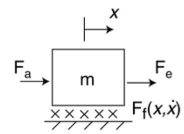
- **Feather-light touch** at forces up to and beyond body weight
  - High force density (force/mass ratio)
  - Low output (driving-point) mechanical impedance



- Present Actuator Technologies
  - Electromagnetic: low force density
  - Hydraulic, geared electromagnetic: high intrinsic impedance
  - Compressed-gas: limited by low-frequency “parasitic” dynamics

The Appeal of Force Feedback

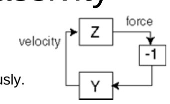
- Equation of motion: $m\ddot{x} + F_f(x,\dot{x}) = F_a + F_e$
- Force feedback controller: $F_a = G_f F_e$
- Resulting equation of motion: $\frac{m\ddot{x}}{1+G_f} + \frac{F_f(x,\dot{x})}{1+G_f} = F_e$
- Increasing G_f reduces apparent inertia, friction



SNAG—Coupled Instability

Coupled Stability via Passivity

- *A passive* impedance has Z(s) positive real
  - Phase of Z(s) lies between +90° and -90°
  - System may store, dissipate & return energy—
  - but cannot be “pumped” to supply power continuously.



- Physical interaction resembles unity negative feedback
  - Couple two passive systems
    - Combined phase lies between +180° and -180°⇒ STABLE
    - No constraint on magnitude
- Controller design constraint:
  - Imposing passive robot impedance guarantees stability when coupled to *all* passive objects.
    - Arbitrarily complicated collections of springs, masses, dampers, constraints, etc.

Hogan, N. (1988) *On the Stability of Manipulators Performing Contact Tasks*, IEEE Journal of Robotics and Automation, 4: 677-686.
Colgate, J. E. and Hogan, N. (1988) *Robust Control of Dynamically Interacting Systems*, International Journal of Control, Vol. 48, No. 1, pp. 65-88.

Force Feedback and Passivity

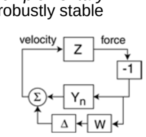
- Passivity is hard to achieve
  - Discrete-time implementation exceeds phase constraint at high frequencies
  - High-gain force feedback with resonant dynamics between sensor & actuator violates passivity
- With force feedback passivity is conservative
  - With *any* resonant dynamics between sensor & actuator, force feedback inertia reduction by 50% or more is non-passive [Colgate ’89]
  - Severely limits force feedback loop gain

Complementary Stability

- Define a bounded set of environment port operators:

$$Y(s) = Y_n(s) + W(s)\Delta(s) \qquad \|\Delta(s)\|_\infty \le 1$$

- Definition: A robot represented by *Z* achieves *complementary stability* with the set *Y* if the coupled system is robustly stable
  - Stability analysis by the small gain theorem
  - Additive perturbation structure is not essential

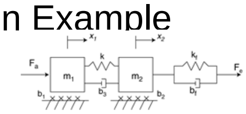


Controller Design via Constrained Optimization

- Prerequisites
  - Model of robot (with at least one resonance)
  - Model (or data representation) of environment port admittance
  - Assumed controller structure with selected variable parameters
- Algorithm
  - Broad search finds parameter combinations to satisfy complementary stability
  - Select best-performing stable controller(s) based on robot impedance magnitude

Controller Design Example

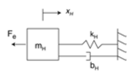
- Robot model:
  - Single-resonance, with force transducer



- Control structure:
  - Vary *p, z,* K_{dc}
  - Target impedance *Z=0*

$$F_a = K_{dc}\frac{p}{z}\frac{(s+z)}{(s+p)}F_e$$

- Environment model:

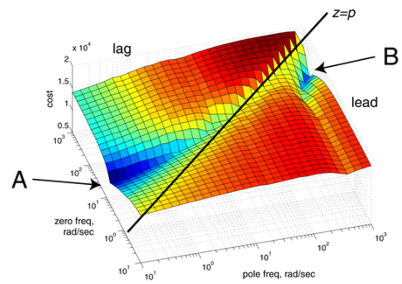


- Stability by structured singular value
- Performance "cost":

$$C = \sum_{\omega_0}^{\omega_1} \log|Z(j\omega)|$$

- Parameters based on laboratory robot module, literature on human arm endpoint dynamics

Example Results

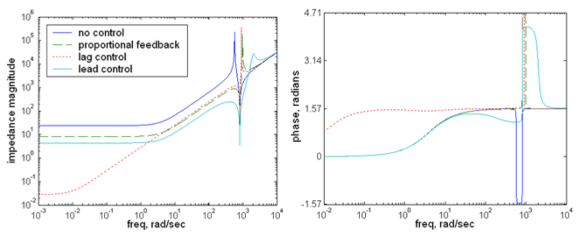


Cost *C* at maximum stable K_{dc} versus *p* and *z*

Region "A" indicates low-frequency lag control

Region "B" indicates high-frequency lead control

Example Results (continued)



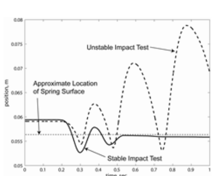
Algorithm returns non-obvious controller parameters.

Implementation

- Apply control to physical system
  - Screw-driven robot module
  - 140 N continuous force capacity
  - Up to 20 N Coulomb friction, position dependent
  - Approximately 6 kg endpoint inertia
  - High-frequency noise in force sensor precludes high-frequency (lead) control
- Model is linear, robot significantly not
  - Robust testbed for control approach

Stability

- Contact tests with spring (and plastic block) environments



Increasing Environment Stiffness →

	none	1	2	3	4	5	6	block	arm
Kdc=850							PXO	++	+
Kdc=2000	P					X	O	++	+
Kdc=3000	P				X		O	++	+
Kdc=5000	P				X		O	+-	-

Controller specified by algorithm (green)
Explicitly part of design set (yellow)

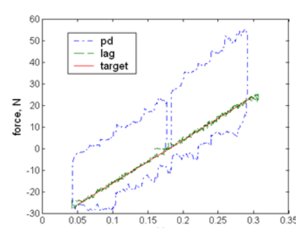
X Maximum stiffness stable in impact test
O Maximum stiffness stable in contact
P Predicted maximum stable stiffness

Last column indicates behavior coupled to human arm

"-" indicates unwanted vibration

Model-based algorithm results are more conservative than experiments, less conservative than passivity

Achieved Performance



Performance best tested by "feel"

Static/Coulomb friction:

less than 0.3 N (66x reduction)

Inertia reduction:

1.75 kg with K_{dc}=2000 (3.4x reduction)

1.2 kg with Kdc=3000 (5x reduction)

Performance and stability are significantly enhanced

—despite differences between model and robot.

Passivity

[redacted]

- A powerful tool for sophisticated controller design
  - Nonlinear, Adaptive, Robust
  - Fundamentally a ***physical system*** concept
- Passive interactive behavior guarantees contact stability
  - Drawback: conservative for force feedback
- Reduce conservativeness:
  - Optimize for interaction with a limited set of objects
  - —e.g. humans

[redacted]

Buerger, S.P. and Hogan, N. *Complementary Stability and Loop-Shaping for Improved Human-Robot Interaction*. IEEE Transactions on Robotics 2007 23:232-244

Robot Impedance Control

- Biologically-inspired approach to interaction control
  - Controller establishes a *relation* between force and motion (Hogan, 1980, 1985, ...)
  - Two implementations:
    - impedance (force-out-for-motion-in)
      - like "frequency-dependent" stiffness
    - admittance (motion-out-for-force-in)
      - like "frequency-dependent" inertia
    - controlling robot impedance is ideal, controlling robot admittance is often easier
- Works well for interaction tasks:
  - Automotive assembly (Case Western Reserve University, US)
  - Food packaging (Technical University Delft, NL)
  - Hazardous material handling (Oak Ridge National Labs, US)
  - Automated excavation (University of Sydney, Australia)
  - … and ***many*** more

Human-Robot Cooperation

Toyota Motor Corporation Vehicle Assembly

Biomimetic Artificial Arms

- Application: Motorized artificial arms for amputees
- Goal: Provide more natural function
  - Most arm amputees are uni-lateral, with an unimpaired arm
    - Dexterous artificial hands are a *tough* technical challenge
      - No practical solutions are yet available
    - Fine-motor tasks are performed with the unimpaired hand
    - The prosthetic arm serves the non-dominant role
      - Support objects, steady them, etc.
  - Hence skillful control of physical contact is the key to arm prosthesis function
    - Arm-waving is not enough!
- Innovation: A biomimetic arm prosthesis with controllable mechanical impedance

Biomimetic Motorized Prosthetic Arms

- Two-way interaction:
  - With the amputee's intact limb segments
  - With objects (s)he manipulates
- Controlling interaction is essential

Dexterity Requires Interaction Control

- A prosthesis must control two essential physical interactions:
  - With the world
    - The key to functional tasks
  - With the natural limbs
    - The key to coordination
- Conventional motion control doesn't work
  - Among many reasons, it requires excessive information and control precision
- Impedance control* can mimic natural arm control
  - Enables multi-joint coordination and coping with difficult mechanical constraints

*Hogan, 1980, 1985 etc. Hogan & Buerger, 2005 …

“Natural” Impedance Control Details

- Programmable prosthesis “emulator”
  - A “flight simulator” for amputation prostheses
  - Enables precisely controlled study of alternatives
- Trans-humeral (above-elbow) amputation in these studies
  - Motorized elbow, cable-operated terminal device
    - Conventional two-bladed hook
  - Command and control interface: EMG from residual biceps & triceps
    - Electrodes built into socket over limb residuum
    - Optimal EMG magnitude estimation*
- “Natural” Impedance Control mimics natural muscle behavior
  - Differential muscle contraction determines elbow equilibrium position
    - Highly “back-drivable” dynamics allows displacement from equilibrium
    - Responsive to external forces or from other body segments
  - Antagonist co-contraction determines impedance
    - Elbow stiffness & damping increase with sum of EMG magnitudes
    - Enables stable interaction, load-bearing, force transmission

*Clancy & Hogan 1995, 1997

Enables Functional Bi-Manual Tasks

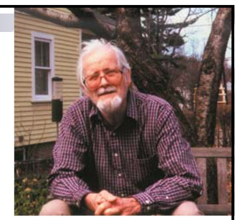
- Key features of “natural impedance control”
  - Controlled deflection under load similar to natural elbow facilitates
    - Bi-manual coordination
    - Production of useful work
  - Guaranteed stability during physical interaction facilitates
    - “Carefree” interaction with other body parts
      - The other hand, the foot, the thigh …

Implications of Natural Impedance Control for Brain-Machine Interfaces

- Simplifies the human-machine interface
  - **Reduces** the precision and channel capacity required for command and control
- Readily scales to many degrees of freedom
  - Impedance control is a key feature of
    - NASA/GM’s Robonaut 2
    - Toyota’s vehicle assembly
    - Kuka/DLR’s “lightweight arm”
      - …and many more
- Easily controls “excess” (redundant) degrees of freedom
  - Position of the hand and impedance of the joints is sufficient to achieve smooth coordination*
- Requirement: Sufficiently “back-drivable” hardware is **essential**
- Next step: Replace EMG with EEG
  - … or whatever interface has the required capacity

*Mussa-Ivaldi & Hogan 1991

Physical System Theory



Henry Martyn Paynter
MIT Mechanical Engineering Faculty
ASME Rufus Oldenburger Medalist

- Is there a *general* theory of physical systems?
  - Rich history: Maxwell (1873) Firestone (1933) Trent (1955) …
- Most comprehensive: Paynter (1961)
  - Elaborated by many, notably Peter Breedveld, U. Twente, Netherlands
- Key concept: Interaction port
  - Every distinct way power can be exchanged defines a port
- Identifying ports enables ***reticulation***
  - Forming a network of interconnected pieces
    - This is what Kron (1936) called *diakoptics* (Greek for “tearing”)
- Reticulation may proceed until each piece is an idealized element…
  - The familiar mass, spring, damper, resistor capacitor inductor, etc
- …but that’s not essential
  - **Network models are *not* restricted to “lumped-parameter” models**

Network Physical System Models

- Network models are versatile
  - The pieces connected may be nonlinear and multivariable
  - They readily yield state-determined representations
    - The basis of (almost) all advanced controller designs $\dot{\mathbf{x}} = f(\mathbf{x}, t)$
- Network physical system models are highly structured
- Remarkably, a symmetric multi-port connection ***must be linear***
  - Even though the connected pieces / elements may be nonlinear*
- This has far-reaching consequences
  - e.g. Tellegen’s theorem
    - Force-like variables (efforts) and motion-like variables (flows) occupy orthogonal sub-spaces
    - ***Completely*** independent of the network elements

*Proof:

Hogan N. *Modularity And Causality In Physical System Modeling*. Journal Of Dynamic Systems Measurement And Control 109: 384-391, 1987.

Paynter HM, and Busch-Vishniac IJ. *Wave-scattering Approaches to Conservation and Causality*. Journal of the Franklin Institute 325: 295-313, 1988.

Network Model Power Ports

- Two real-valued variables quantify power flow between two subsystems
  - Define *u* to quantify power flow out of *A*
    - Square it to ensure sign-definite power flow
  - Define *v* to quantify power out of *B*
    - Square it to ensure sign-definite power flow
  - Net power is a difference of squares
    - Derivation motivated by wave variables but not restricted to wave transmission
  - These variables define a ***power port***

$$P_{A,out} = u^2 \qquad u \in \Re \therefore P_{A,out} \geq 0$$

$$P_{B,out} = v^2 \qquad v \in \Re \therefore P_{B,out} \geq 0$$

$$P_{net,A \to B} = P_{A,out} - P_{B,out} = u^2 - v^2$$

- Power flow may always be expressed as a *product* of two real-valued variables
  - Force, velocity; voltage, current; effort, flow; etc.
    - *c* is a domain-dependent scaling constant
- …but that’s not essential

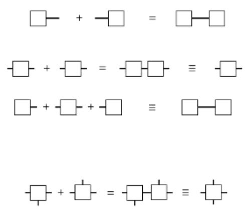
$$P_{net,A \to B} = c(u-v) \cdot (u+v)/c$$

$$e \equiv c(u-v)$$

$$f \equiv (u+v)/c$$

Network Model Connectors

- With only 1-port elements only two may be connected
- Combining 2-port elements yields another 2-port
  - Only linear chains terminated by 1-ports may be connected
- 3-port elements are *necessary* for general networks
- They are also *sufficient*
  - Combinations of 3-port elements yield
    - 4-port elements
    - n-port elements



Three-Port Connectors

- Two vectors of 3 real-valued variables characterize power flow into and out of a 3-port connector
- Power flows are the squares of the lengths of these vectors
- The connector equations are an algebraic relation between these input and output variables

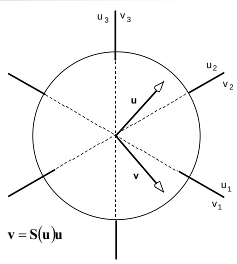
$$\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \qquad \mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$P_{in} = \sum_{i=1}^{3} u_i^2 = \mathbf{u}^t\mathbf{u} \qquad P_{out} = \sum_{i=1}^{3} v_i^2 = \mathbf{v}^t\mathbf{v}$$

$$\mathbf{v} = f(\mathbf{u})$$

Power Continuity

- An ideal connector is power-continuous
  - Power in = power out means the lengths of **u** and **v** must be equal
- The algebraic relation $\mathbf{v} = f(\mathbf{u})$ is equivalent to a rotation operator
- The rotation matrix is orthogonal
  - Rows (or columns) are
    - Orthogonal
    - Unit magnitude



$$\mathbf{v} = \mathbf{S}(\mathbf{u})\mathbf{u}$$

$$\mathbf{v}^t\mathbf{v} = \mathbf{u}^t\mathbf{S}^t\mathbf{S}\mathbf{u} = \mathbf{u}^t\mathbf{u}$$

$$\mathbf{S}^t\mathbf{S} = \mathbf{1}$$

Symmetric 3-port Connectors

- Assume invariance under permutation
  - exchanging any two ports doesn't matter
- Orthogonality of **S** yields
  - two equations for *a* an *b*
  - with only two solutions
  - *a* and *b* must be constant
- There are only two symmetric, power-continuous, three-port connectors
- ***Both are linear***
  - Independent of the (non)linearity of the systems they connect

$$\mathbf{S} = \begin{bmatrix} a & b & b \\ b & a & b \\ b & b & a \end{bmatrix}$$

$$a(u_1,u_2,u_3)^2 + 2b(u_1,u_2,u_3)^2 = 1$$
$$b(u_1,u_2,u_3)^2 + 2a(u_1,u_2,u_3)b(u_1,u_2,u_3) = 0$$

$$a(u_1,u_2,u_3) = 1/3 \qquad b(u_1,u_2,u_3) = -2/3$$

$$a(u_1,u_2,u_3) = -1/3 \qquad b(u_1,u_2,u_3) = 2/3$$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1/3 & -2/3 & -2/3 \\ -2/3 & 1/3 & -2/3 \\ -2/3 & -2/3 & 1/3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \qquad \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -1/3 & 2/3 & 2/3 \\ 2/3 & -1/3 & 2/3 \\ 2/3 & 2/3 & -1/3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

Physical & Computational Dynamics Combined

- Controlled system dynamics emerge from physical dynamics *and* signal processing
  - A central challenge of motor neuroscience:
    - How much behavior is due to (bio-)mechanics?
    - How much is due to (neural) information processing?
- Need: a general theory of actuators
  - signal-to-energy interfaces
    - with
- Two **unambiguously distinct** components
  - "forward path" dynamics
    - Signals influence physical events
  - Interactive dynamics
    - Physical events influence the response to signals

Video courtesy of Prof. Andy Ruina, Cornell University

Equivalent Networks

- Equivalent circuits:
    - Helmholtz (1853) Thevenin (1883) Heaviside (1925) Mayer (1926) Norton (1926)
      - Mayer & Norton published in the same *month* of 1926!
- Two key benefits
  - **Prodigious simplification**
    - Arbitrarily complex systems: simpler functionally equivalent form
  - Unambiguously identifiable
    - Both components experimentally measurable
- For linear systems, equivalent networks serve as interface elements
  - "source" element describes "forward path" dynamics
    - **Independent** of interaction
  - Equivalent "resistance" describes interactive dynamics
    - **Independent** of forward-path dynamics

Nonlinear Equivalent Networks?

- Can equivalent networks be extended to nonlinear systems?
    - Don't anticipate a complete correspondence—but linear network topology encourages optimism
  - Can nonlinear equivalent networks be defined?
  - Can their components be identified unambiguously?
- Linear equivalent network: four possible forms, largely interchangeable
  - Two types of connection
    - Common motion (Helmholtz/Thevenin) common effort (Mayer/Norton)
  - Two operational forms
    - Impedance (motion in/force out) admittance (force in/motion out)
- Nonlinear system:
  - Some forms may not be well-defined
    - e.g. mechanical linkages are "naturally" admittances
  - Unambiguous identification precludes some types

Neuro-Muscular Actuators

$$\mathbf{I}(\boldsymbol{\theta})\dot{\boldsymbol{\omega}} + \mathbf{C}(\boldsymbol{\theta},\boldsymbol{\omega}) = \sum_i \boldsymbol{\tau}_i$$

- Human skeleton
  - Inertial mechanics *comes with* a linear common-motion connection
    - Helmholtz/Thevenin network, operational form: admittance
- Human neuro-muscular system
  - Operational form can't be admittance
    - Non-monotonic force-length curve—same force at different lengths
  - Impedance form is compatible with skeletal admittance
- Which network type?
  - The "obvious" choice: a "force source" modified by interactive dynamics
    - common-motion (Helmholtz/Thevenin) network
  - This assumption is ubiquitous in computational motor neuroscience
    - The forward path specifies nominal muscular forces
- Snag: Interactive dynamics **cannot** be identified unambiguously

Identifying Equivalent Networks

- Observation only from the contact point (interaction port)
  - To identify the source term, enforce zero power exchange
    - Zero motion—i.e. immobilize the (neuro-muscular) actuator
  - Problem: steady-state force due to interactive dynamics
    - If zero, can't stabilize the skeleton
    - If non-zero, can't be distinguished from "source" force
- Alternative: a "motion source" modified by interactive dynamics
    - Common-effort (Mayer/Norton) network
  - The forward path specifies a nominal trajectory and/or posture
  - Interactive dynamics operate on deviations from nominal motion
- Identification:
  - If the interactive dynamics are *locally observable*, Identically zero force identifies the motion source—**unambiguously**
- The biological actuator requires a Mayer/Norton network

Research Required

- The idea of "putting physics in control" is not at all new
  - But a practical, systematic approach has not been fully articulated
- A general physical system theory remains elusive
  - Establishing a solid mathematical foundation is important
  - Differential geometry seems promising
    - e.g. ongoing work on a *port-controlled Hamiltonian* formulation
  - But there's a tradeoff:

Mathematical rigor vs. intuitive comprehension?

A tough challenge!